# Examples of Tasks from CCSS Edition Course 3, Unit 5

**Getting Started**
The tasks below are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing student understanding and independence. A list of questions to use as you work together, prepared in English and Spanish, is available. Encourage students to refer to their class notes and Math Toolkit entries for assistance. Comments in red type are not part of the solution.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See Research on Communication.

The Algebra and Functions page might help you follow the conceptual development of the ideas you see in these examples.

**Main Mathematical Goals for Unit 5**
Upon completion of this unit, students should be able to:

- recognize patterns in problem conditions and in data plots that can be described by polynomial and rational functions.
- write polynomial and rational function rules to describe patterns in graphs, numerical data, and problem conditions.
- use table, graph, or symbolic representations of polynomial and rational functions to answer questions about the situations they represent: (1) calculate $y$ for a given $x$ (i.e., evaluate functions); (2) find $x$ for a given $y$ (i.e., solve equations and inequalities); and (3) identify local max/min points and asymptotes.
- rewrite polynomial and rational expressions in equivalent forms by expanding or factoring, by combining like terms, and by removing common factors in numerator and denominator of rational expressions.
- add, subtract, and multiply polynomial and rational expressions and functions
- extend understanding and skill in work with quadratic functions to include completing the square, interpreting vertex form, and proving the quadratic formula.
- recognize and calculate complex number solutions of quadratic equations.

**What Solutions are Available?**



**Lesson 3:** Investigation 1—Applications Task 4 (p. 381)
Investigation 2—Applications Task 5 (p. 382), Applications Task 7 (p. 382), Connections Task 17 (p. 384), Extensions Task 24 (p. 386)
Investigation 3—Extensions Task 25 (p. 389), Review Task 37 (p. 389)
Investigation 4—Applications Task 10 (p. 382), Reflections Task 23 (p. 385), Review Task 40 (p. 389)

## Selected Homework Tasks and Expected Solutions

(These solutions are for tasks in the CCSS Edition book.
For homework tasks in books with earlier copyright dates, see Helping with Homework.)

### Lesson 1, Investigation 1, Applications Task 1 (p. 336)

**a–c.** To be completed by the student.

**d.** Points used: (–0.5, 0), (0, –4), (1, –5), (2, –5), (3, –4), (3.5, 0)
$y = 0.476x^4 - 2.857x^3 + 5.738x^2 - 4.357x - 4$; Since this graph is flatter near the minimum value, a quadratic function is not reasonable. But a function with even degree is a good choice.

### Lesson 1, Investigation 1, Review Task 24 (p. 345)

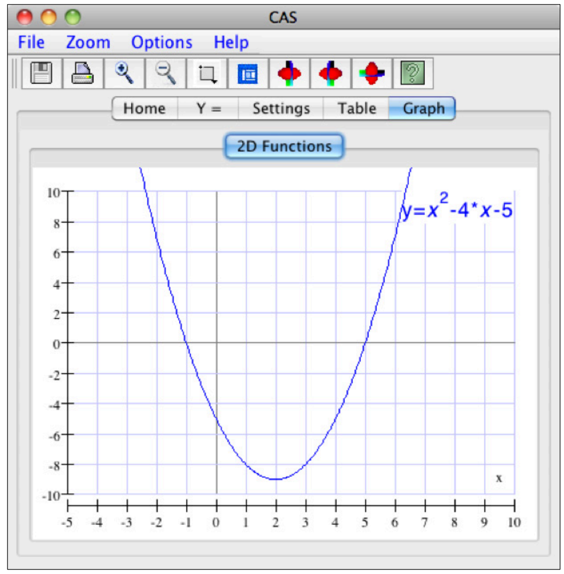
In Course 3 Unit 2, *Inequalities and Linear Programming*, students learned to solve quadratic inequalities by factoring the inequality set equal to zero. Then they used the zeroes of the linear terms or corresponding functions to make a sketch and determine the solution. This method is shown in Part c.

**a, b, d.** To be completed by the student.

**c.**

| | |
|---|---|
| $x^2 - 4x - 5 \geq 0$ | Original inequality |
| $x^2 - 4x - 5 = 0$ | Change the inequality to an equality to find the zeroes (the $x$-intercepts) of the function. |
| $(x - 5)(x + 1) = 0$ | Factor the non-zero side. |
| $x - 5 = 0$ or $x + 1 = 0$<br>$x = 5$ or $x = -1$ | Set each factor equal to zero and solve to get the zeroes of your equation. Use these with other information in the equation to graph the function. |

Use the information from the graph of the corresponding function to determine your solution. The function is greater than or equal to zero when $x \le -1$ or $x \ge 5$, so these intervals are the solution to the inequality $x^2 - 4x - 5 \ge 0$.




## Lesson 1, Investigation 2, Applications Task 3 (p. 337)

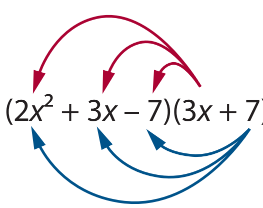
**a–d.** To be completed by the student.

**e.** $5x^5 - 3x^4 + 7x^3 + 3x - 2$
The degrees of the expressions being combined are 5 and 3. The degree of the result is 5.

## Lesson 1, Investigation 2, Applications Task 5 (p. 338)

**a, b, d–f.** To be completed by the student.

**c.** To multiply $(2x^2 + 3x - 7)(3x + 7)$, you can use the distributive property twice.



$$(2x^2 + 3x - 7)(3x + 7) = (3x)(2x^2) + (3x)(3x) + (3x)(-7) + (7)(2x^2) + (7)(3x) + (7)(-7)$$
$$= 6x^3 + 9x^2 - 21x + 14x^2 + 21x - 49$$
$$= 6x^3 + 23x^2 - 49$$

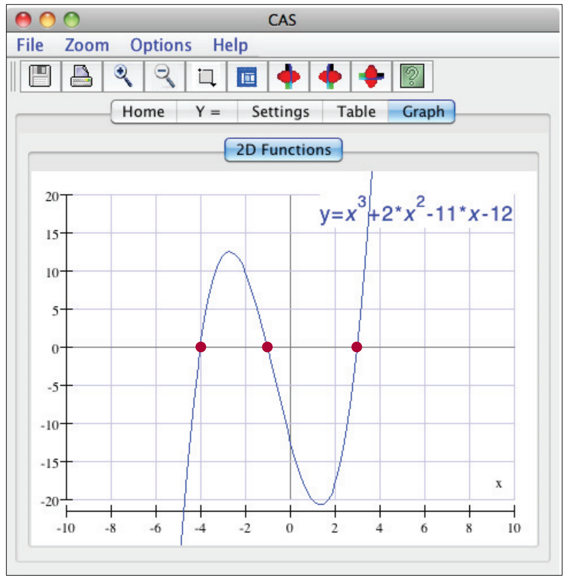
## Lesson 1, Investigation 2, Connections Task 13 (p. 341)

**a.** To solve polynomial inequalities, you must find the zeroes of the corresponding polynomial function. The zeroes can be easily found by looking at the factored form, which is given to us in the problem.

$$y = x^3 + 2x^2 - 11x - 12 = (x - 3)(x + 1)(x + 4)$$

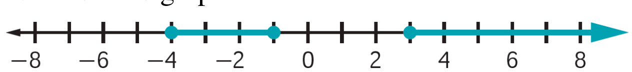
The zeroes are 3, –1, and –4. To solve $x^3 + 2x^2 - 11x - 12 \ge 0$, one needs to look at the graph of the corresponding function as shown at the top of the following page.




By looking at the graph, we can see the function is greater than zero for $x$ values between –4 and –1. It is also greater than zero for $x$ values larger than 3. The solution to $x^3 + 2x^2 - 11x - 12 \geq 0$ can be expressed three ways:

- Inequality notation: $-4 \leq x \leq -1$ or $x \geq 3$
- Interval notation: $[-4, -1] \cup [3, \infty)$
- Number line graph:




**b.** To be completed by the student.

## Lesson 1, Investigation 2, Review Task 28 (p. 346)

**a.** $\sqrt{24} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$ ; The number 24 has many more factors like 2(12) or 3(8), etc. So, why was 4(6) chosen? The reason is the 4 is what is called a perfect square ($2^2 = 4$), so $\sqrt{4} = 2$. Thus, when putting the expression in simplest equivalent form, look for factors that are perfect squares.

**b.** $\sqrt{48} = \sqrt{4}\sqrt{12}$
$= \sqrt{4}\sqrt{4}\sqrt{3}$
$= 2 \bullet 2 \bullet \sqrt{3}$
$= 4\sqrt{3}$

or

$\sqrt{48} = \sqrt{16}\sqrt{3}$
$= 4\sqrt{3}$

**c–h.** To be completed by the student.

## Lesson 1, Investigation 3, Applications Task 8 (p. 339)

**a.** Recall *profit* = *income* – *expenses*.
*income* = (*number sold*)(*price*) = $(100 - 4x)(x) = 100x - 4x^2$ (from Task 7) and *expenses* = $2x + 150$. So, one algebraic expression for *profit* is $P(x) = (100x - 4x^2) - (2x + 150)$. The other simpler equivalent form is left for the student.

**b–f.** To be completed by the student.

## Lesson 1, Investigation 3, Reflections Task 16 (p. 342)

The answers to the following questions are for the function:
$f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24 = (x - 1)(x - 2)(x - 3)(x - 4)$

**a.** It is easier to see the zeroes in the factored form. You find them by setting each linear factor equal to zero and solving. $f(x) = (x - 1)(x - 2)(x - 3)(x - 4)$

| | | | |
|---|---|---|---|
| $x - 1 = 0$ | $x - 2 = 0$ | $x - 3 = 0$ | $x - 4 = 0$ |
| $x = 1$ | $x = 2$ | $x = 3$ | $x = 4$ |

**b.** It is easier to see the $y$-intercept in the expanded standard form. Since the $y$-intercept has an $x$-coordinate $x = 0$ and each term of the polynomial function except the constant term is a power of $x$ times a constant, $f(0) = 24$. The $y$-intercept is $(0, 24)$.

**c–e.** To be completed by the student.

## Lesson 1, Investigation 3, Review Task 30 (p. 346)

**a.** $x = \pm 4$

**b, c, e, f.** To be completed by the student.

**d.** $x = -3 \pm \sqrt{19}$

## Lesson 1, Investigation 3, Review Task 31 (p. 346)

**a.** $4x^2 + 17x - 15$

**b, d–f.** To be completed by the student.

**c.** $x^2 - 9$

## Lesson 2, Investigation 1, Applications Task 1 (p. 357)

**a.** $f(x) = (x + 2)^2 - 9$

**Max/min points:** This particular function has a minimum point and it occurs at $(-2, 9)$. For any quadratic function in the vertex form $f(x) = a(x - h)^2 + k$, the vertex (max or min) is the point $(h, k)$. This idea was developed in the investigation in Problem 2 on page 349. It should also be written in the student's math toolkit.

***x*-intercepts:** $(-5, 0)$ and $(1, 0)$
To find the $x$-intercepts, you let $f(x) = 0$ and solve.
$(x + 2)^2 - 9 = 0$
$(x + 2)^2 = 9$
$(x + 2) = \pm 3$
$x = -5$ or $x = 1$

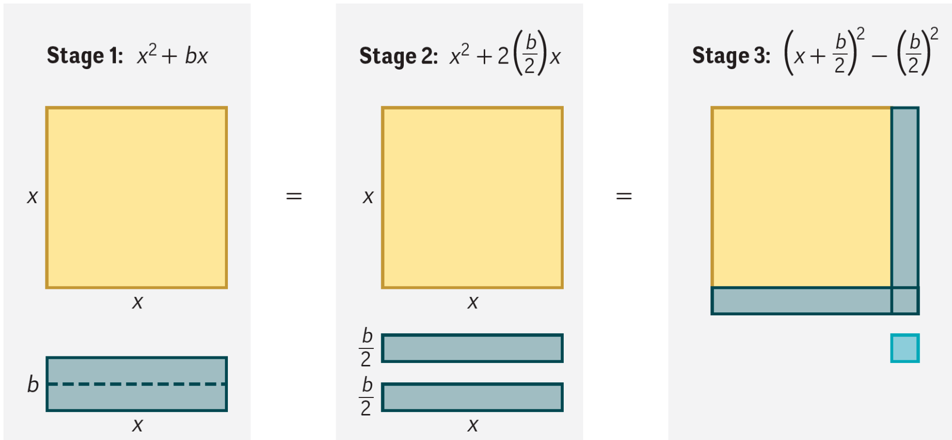
***y*-intercept:** $(0, -5)$
Find $f(0) = (0 + 2)^2 - 9 = 2^2 - 9 = 4 - 9 = -5$.

**b–d.** To be completed by the student.

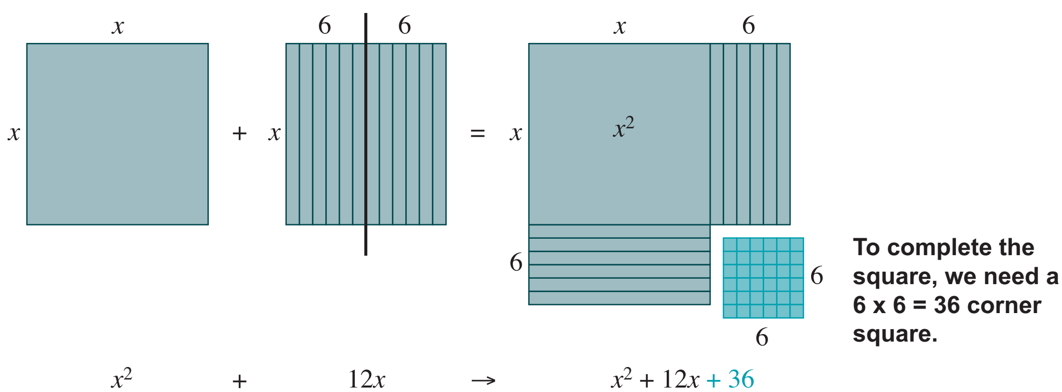
 

## Lesson 2, Investigation 1, Applications Task 4 (p. 358)

These problems ask you to write a rule in vertex form. (See page 350.) To do this, you follow a procedure called completing the square.




**a.** $f(x) = x^2 + 12x + 11$
For the time being, just put the +11 to the side. We will deal with it later. So, just consider $f(x) = x^2 + 12x$. It helps to turn this into a picture.




Now the square is complete. The function now looks like $f(x) = x^2 + 12x + 36$. This is a perfect square trinomial which factors into $f(x) = (x + 6)^2$. Now what to do with the 11?

The function $f(x) = x^2 + 12x + 36 + 11 - 36$ is obviously equivalent to the original function. So, we can factor the perfect square trinomial and combine the constant terms that are left to get this: $f(x) = (x + 6)^2 - 25$. Once the function is in this form, the maximum point is easily identified, by the point (–6, –25).

**b–d.** To be completed by the student.

## Lesson 2, Investigation 1, Applications Task 5 (p. 358)

**a.** $x^2 + 12x + 11 = 0$. We changed the form of the function by completing the square in Task 4 to get this new form $(x + 6)^2 - 25 = 0$.

$(x + 6)^2 - 25 = 0$
$(x + 6)^2 = 25$
$x + 6 = \pm\sqrt{25}$
$x = -6 \pm 5$
$x = -1$ or $x = -11$

**b–d.** To be completed by the student

## Lesson 2, Investigation 1, Extensions Task 21 (p. 361)

Use your experience in Task 20 to solve these quadratic equations by first writing the quadratic expression in equivalent form as a product of two linear factors.

**a.** $2x^2 + 7x + 3 = 0$
$(2x + 1)(x + 3) = 0$
$2x + 1 = 0$ or $x + 3 = 0$
$2x = -1$     $x = -3$
$x = -\frac{1}{2}$

**b–f.** To be completed by the student

## Lesson 2, Investigation 1, Review Task 26 (p. 362)

**a.** $\frac{1}{2}$

**b.** $\frac{3}{8}$

**c–e.** To be completed by the student

## Lesson 2, Investigation 2, Applications Task 6 (p. 358)

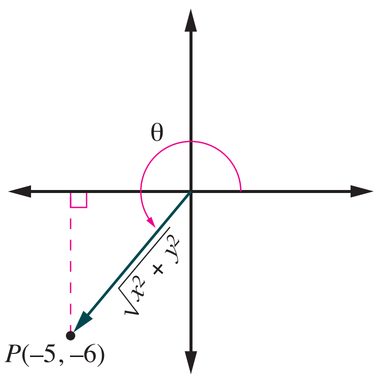
**a.** $x = 1, x = -\frac{5}{2}$; rational

**b.** $x = 1, x = -\frac{3}{2}$; rational

**c–f.** To be completed by the student

### Lesson 2, Investigation 2, Review Task 32 (p. 363)

Students should have a copy of Selected Key Geometric Ideas from Courses 1 and 2 and/or their Math Toolkit notes as reference if they do not recall ideas being reviewed, such as the definitions of trigonometric functions on $0^\circ \le \theta \le 360^\circ$. Students should be developing the habit of looking up ideas that they may not remember.




$\sin\theta = -\frac{6}{?}$

$\cos\theta = -\frac{5}{?}$

$\tan\theta = \frac{?}{?}$

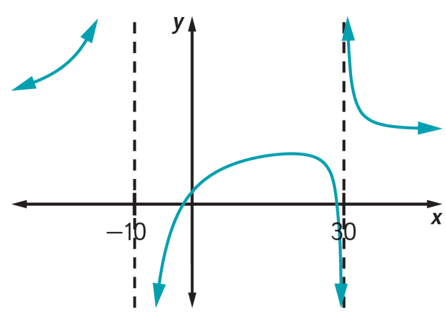
### Lesson 3, Investigation 1, Applications Task 4 (p. 381)

**a.** One rule that shows the separate expressions for calculating income is $P(x) = (-25x^2 + 500x + 7{,}500) - (7{,}000 - 200x)$.
The second rule is to be found by the student.

**b.** **i.** $\frac{P(x)}{I(x)} = \frac{-25x^2 + 700x + 500}{-25x^2 + 500x + 7{,}500}$

- Ratio of profit to income
- The expression can be evaluated for all $x \neq -10, 30$. It makes sense to consider a domain of $0 < x < 30$.
- There are vertical asymptotes at $x = -10$ and at $x = 30$.

The remainder of Part b to be completed by the student.




### Lesson 3, Investigation 2, Applications Task 5 (p. 382)

$\frac{P(x)}{I(x)} = \frac{-25x^2 + 700x + 500}{-25x^2 + 500x + 7{,}500} = \frac{-25(x^2 - 28x - 20)}{-25(x^2 - 20x - 300)} = \frac{x^2 - 28x - 20}{(x + 10)(x - 30)}$ (domain is unchanged)

The remainder of the task is to be completed by the student.

### Lesson 3, Investigation 2, Applications Task 7 (p. 382)

**a.** $\frac{4x + 12}{8x + 4} = \frac{4(x + 3)}{4(2x + 1)} = \frac{x + 3}{2x + 1}$; Both expressions are undefined when $x = \frac{1}{2}$.

**b–d.** To be completed by the student.

### Lesson 3, Investigation 2, Connections Task 17 (p. 384)

**a.** Remember that asymptotes are equations of lines. The asymptotes are at $s = 0$ (vertical) and $T(s) = 0$ (horizontal).

**b.** To be completed by the student.

### Lesson 3, Investigation 2, Extensions Task 24 (p. 386)

**a.** $f(x)$ is undefined when $x = 3$ and when $x = –2$.

**b, d, e.** To be completed by the student.

**c.** It is undefined only when $x = –2$.

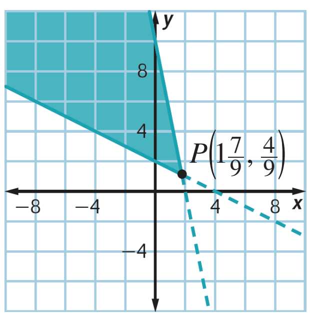
### Lesson 3, Investigation 3, Extensions Task 25 (p. 386)

**a.** Essential discontinuity at $x = –0.5$; no removable discontinuities

**b–d.** To be completed by the student.

### Lesson 3, Investigation 3, Review Task 37 (p. 389)

**a.**




The solution is the coordinates of all points enclosed by the solid rays extending from the point $\left(1\frac{7}{9}, \frac{4}{9}\right)$ and the points on these rays.

**b.** To be completed by the student.

### Lesson 3, Investigation 4, Applications Task 10 (p. 382)

**a.** $\dfrac{2x + 4}{x^2 - 6x} \cdot \dfrac{x^2 - 36}{4x + 8} = \dfrac{2(x + 2)(x + 6)(x - 6)}{x(x - 6)4(x + 2)} = \dfrac{x + 6}{2x}$

**b–d.** To be completed by the student.

## Lesson 3, Investigation 4, Reflections Task 23 (p. 385)

**a.** Vertical asymptotes will occur only when the denominator is 0. Even when the denominator is zero, some discontinuities of a rational function may not be clearly visible in a graph when the zero of the denominator is also a zero of the numerator.

**b–d.** To be completed by the student.

## Lesson 3, Investigation 4, Review Task 40 (p. 389)

**a.** $\triangle ABC \cong \triangle DEF$ (HA or ASA)

**b, c.** To be completed by the student.