Examples of Tasks from Course 1, Unit 5

What Solutions are Available?

Lesson 1: page 335, Modeling Task 3; page 336, Organizing Task 1; page 347, Modeling Task 3; page 348, Organizing Task 1; page 352, Extending Task 1

Lesson 2: page 367, Modeling Task 4; page 369, Organizing Task 3; page 371, Reflecting Task 4; page 377, Modeling Task 1; page 378, Modeling Task 4; page 379, Organizing Task 1; page 379, Organizing Task 2; page 381, Extending Task 1; page 397, Modeling Task 5; page 397, Organizing Task 1

Lesson 3: page 408, Modeling Task 1; page 411, Organizing Task 3

These tasks are selected with the intent of presenting key ideas and skills. Not every answer is complete, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing your child’s understanding and independence.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See Research on Communication.

The Geometry page or the Scope and Sequence might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 5

Upon completion of this unit, students should be able to:

• identify, describe, and sketch prisms, pyramids, cones and cylinders
• apply the concepts and computational formulas for perimeter, area, and volume
• use visualization skills to interpret and reason about situations involving two- and three-dimensional shapes, essential in understanding different kinds of symmetry
• classify polygons and analyze their properties (the study of which becomes much more formal in Course 3)
• identify, and explain, different kinds of symmetry for two- and three-dimensional shapes
Selected Homework Tasks and Expected Solutions

(These solutions are for problems in the book with 2003 copyright. If a student is using a book with an earlier copyright, you may notice that some problems may not match exactly, although the intent of the problems are the same.)

Lesson 1, page 335, Modeling Task 3
a. The base of the tower is a square prism. It has a square base and rectangular sides. The next level is a square pyramid with the top cut off. Above that is another square prism. The top is another square pyramid.

b. A tetrahedron would be formed by segments joining the 4 atoms. This shape has 4 vertices, and 4 triangular faces. Any plane of symmetry will divide the shape into two congruent halves. To draw one plane of symmetry, you could join vertex A to vertex B to the center point of the edge opposite to both of these vertices (see example below). This can be repeated with vertices A and C, vertices A and D, vertices B and C, vertices B and D, and vertices C and D. So there are 6 planes of symmetry.

[Diagram showing a tetrahedron with vertices A, B, C, and D]

Developing the ability to visualize is a skill that requires lots of opportunities to practice. Students used hands-on models in class and are asked to make sketches.

c. The base of each pyramid is a square and then the sides slant up so that they meet at a single point at the top of the pyramid directly above the “center” of the square base. Students should find and identify four planes of symmetry.

Lesson 1, page 336, Organizing Task 1
a. 6 faces, 12 edges, 8 vertices

This task uses the NOW-NEXT equation for a relationship in a geometric context. It also emphasizes collecting data and seeking a pattern in the data. These are all ideas that students encountered in Units 1, 2, and 3.
b. Notice that every time a corner is removed it creates a new triangular face, and three new vertices, but removes a vertex. (This answer assumes non-overlapping slices.)

<table>
<thead>
<tr>
<th></th>
<th>Faces</th>
<th>Edges</th>
<th>Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>6</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>First Slice</td>
<td>7</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Second Slice</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third Slice</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fourth Slice</td>
<td>10</td>
<td>24</td>
<td>16</td>
</tr>
</tbody>
</table>

c. For faces, $NEXT = NOW + 1$. For edges, $NEXT = NOW + 3$. For vertices, $NEXT = NOW + 2$.

d. Answer left for student to complete.

Lesson 1, page 347, Modeling Task 3

a. 7

b.  

<table>
<thead>
<tr>
<th>Top</th>
<th>Front</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


c. Many students may find isometric or rectangular dot paper helpful. Constructing a model with cubes could help students develop spatial visualization skills.
d. Notice that these face views are not sufficient to create the unique model pictured above.

Lesson 1, page 348, Organizing Task 1

a. Some shared properties include
   - No edges cross;
   - Neither figure is rigid;
   - Each figure has twelve edges;
   - Each figure has eight vertices.

b. i. ii. iii.

Lesson 1, page 352, Extending Task 1

The net of a square pyramid has to have 4 congruent isosceles triangles, and a square. There are many ways to connect these shapes. Two possible nets are shown at the right. The net on the left requires eight cuts because its perimeter is made of eight segments. The net on the right requires only seven cuts.
Lesson 2, page 367, Modeling Task 4

a. The buckling will occur so that the middle of the 220-foot rail is under the highest point of the buckle.

b. *It is important for students to make conjectures. It doesn’t matter that the conjecture is incorrect; what matters is that students check their conjectures using mathematics that makes sense to them. There is much more ownership in the result when students conjecture first.*

c. There are two figures that are approximately right-angle triangles in the sketch. We know that one side of the triangle measures 110 feet, or 1,320 inches. The hypotenuse of the triangle measures 1,320.6 inches. Using the Pythagorean Theorem, \((1,320.6)^2 = 1,320^2 + h^2\), so \(h = 39.8\) inches.

![Right Triangle Diagram](image)

\(1,320.6\) in. \(h\)

1,320 in.

d. Our estimate is based on modeling the situation with a right triangle. But the rail will actually be curved, so the actual height of the buckle will be slightly less than our result.

e. If a 1.2-inch gap was placed between each section of 220-foot rail then all the expansion could be accommodated horizontally. There would be no buckle.

Lesson 2, page 369, Organizing Task 3

<table>
<thead>
<tr>
<th>Side Length (cm)</th>
<th>Diagonal (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.8</td>
</tr>
<tr>
<td>4</td>
<td>5.7</td>
</tr>
<tr>
<td>7</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This kind of application of fairly simple mathematics, which gives a surprising result, encourages students to think of mathematics as a useful subject, not confined to classrooms and homework assignments.

This task draws on data analysis ideas developed in Units 1, 2, and 3. The data in the table will vary some because students are asked to measure, not calculate, these lengths.
c. Yes, the plot appears to be linear. Students can draw in the line that best fits the points, or use the linear regression line from their calculators.

d. The equation should be approximately \( y = 1.4x \).

   - The slope 1.4 gives the approximate ratio of diagonal to side length. It also means that for every 1-cm increase in side length, the diagonal length increases by about 1.4 cm.
   - The \( y \)-intercept is 0. Technically, there is no square with a side length of 0. However, as the side lengths get closer to 0, the diagonal lengths also get closer to 0.

e. Using the pattern established, the length of the diagonal should be 1.4(55) = 77 cm.

f. Using the Pythagorean Theorem, the diagonal is found from \( h^2 = 55^2 + 55^2 \). \( h \approx 77.78 \approx 78 \) cm. The differences are due to rounding \( \sqrt{2} \) to 1.4 in the linear model.

**Lesson 2, page 371, Reflecting Task 4**

a. The number of regions doubles with each new point added.

b. The maximum number of regions is 31, not 32.

c. One has to be careful when generalizing from only a few data values. Also it is important to verify conjectures using reasoning.

*In Reflecting Task 4, students are encouraged to conjecture and check their conjectures. In fact, the most commonly made conjecture for this example is incorrect, reinforcing the caution one should use when working with only a few data points, and also the importance of verifying conjectures.*
Lesson 2, page 377, Modeling Task 1

a. The volume of the “best buy” is \( \pi (9)^2 (55) \approx 13,996 \) cubic feet. This is not big enough for their needs.

b. Students must solve \( 20,000 = \pi r^2 h \) for \( h \), substituting different values of \( r \). This could be done by solving 6 different equations by hand. Or students might use their calculators to evaluate \( h = \frac{20,000}{\pi r^2} \) for different values of \( r \).

This could be done by entering \( y = \frac{20,000}{\pi x^2} \) and setting up a table that starts at 8 and increases by an increment of 0.5. (The radius is between 8 and 10.5, since the diameter is between 16 and 21.)

c. The silos with 19 and 20 foot diameters will fit on the pad and are not too tall.

Lesson 2, page 378, Modeling Task 4

a. This is a hexagonal prism, the bases being the sides of the pool, and the height being the 18-foot width of the pool. Students should make a labeled sketch (like that which is started below). They will have to use the Pythagorean Theorem to find the length of the sloping edge.
b. The base of the prism is comprised of two rectangles and a triangle, with all dimensions known so students can compute the areas. They should adjust the depth so that at the shallow end depth is 2.5 feet and at the deep end is 8.5 feet. The area of the base is 154 square feet. Then they can figure the volume of water = area of base × height, with the base being the sides of the pool, and height being the 18-foot width.

c. The area to paint includes the side walls, the rectangular walls at the shallow and deep ends, and the bottom of the pool, which is actually 3 more rectangles. When the total area is found, students should determine that they will need to purchase three 5-gallon cans of paint.

- 3 × 18 (shallow end wall) = 54 square feet
- 9 × 18 (deep end wall) = 162 square feet
- (6 × 18) × 2 = 216 square feet
- 17.1 × 18 = 307.8 square feet
- + two side walls
d. 704 square feet
e. 522 six-inch square tiles

Lesson 2, page 379, Organizing Task 1

<table>
<thead>
<tr>
<th>a-b.</th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>60</td>
<td>1 × 1 + 1 × 1 + 1 × 60 + 1 × 60 + 1 × 60 + 1 × 60 = 242</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>30</td>
<td></td>
<td>1 × 2 + 1 × 2 + 1 × 30 + 1 × 30 + 2 × 30 + 2 × 30 = 184</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>12</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c. Students should identify the box with the smallest surface area.

d. The statement is false. All of the above rectangular prisms have the same volume but different surface areas.

Lesson 2, page 379, Organizing Task 2

a. \[ h^2 = s^2 - \left(\frac{s}{2}\right)^2 = s^2 - \frac{s^2}{4} = \frac{3}{4}s^2, \quad h = \sqrt{0.75} \approx 0.8665s. \] So the area is approximately \((0.5)(s)(0.8665s) = 0.4335s^2\) square units.

b. Since a regular hexagon with side length \(s\) can be divided into 6 equilateral triangles, the approximate area of a hexagon is \(6(\text{area of triangle}) \approx 2.6s^2\) square units.

Lesson 2, page 381, Extending Task 1

a. \( \text{Maximum size} = 108 = l + 2a + 2b, \) but in this case the base is square so \(108 = l + 4a, \) where \(a\) is the length of a side of the square and \(l\) is the length of the box (or height of the square prism). If we rewrite this as \(l = 108 - 4a, \) then we can make a table using \(y_1 = 108 - 4x.\) The volume would be \((x^2)l = (x^2)(108 - 4x).\) Defining \(y_2 = (x^2)(108 - 4x)\) produces a table of possible volumes of boxes. The question can be answered from the table.

b. Students have met this idea before. For a given perimeter, a square has greater area than any rectangle. So, using a rectangular prism would not allow for a greater volume of goods.
Lesson 3, page 397, Modeling Task 5

a. The box will be a triangular prism. Students might sketch the box in a variety of ways such as the two shown below.

\begin{center}
\includegraphics[width=0.5\textwidth]{box.png}
\end{center}

b. Any net must show three rectangles with the correct dimensions, and two triangles appropriately connected. There are several choices. (To check their work, students can always draw a net to scale, cut it out, and see if it folds into the correct shape.)

c. \textit{Volume} = \text{base area} \times \text{height} = (0.5)(4)(3)(6) = 36 \text{ cm}^3. \text{ (The situation is made easier by the triangle being a right triangle so the height is given.)}

d. The surface area is made of two triangles and three rectangles. The total surface area is 84 cm².

e. There is only one plane of symmetry because the triangle has no line of symmetry. The plane of symmetry is parallel to the two bases, and cuts the rectangular faces in half.

Lesson 3, page 397, Organizing Task 1

a. Ellen’s rule is correct. Her rule apparently came from thinking of the number of triangles that make up the polygon. From any vertex, one can draw \((n - 3)\) diagonals making \((n - 2)\) triangles, which each contribute 180° to the angle sum. If the polygon is regular, then all angles are equal so this sum is divided by \(n\). The sketch shows the example for a hexagon.
b. As the number of sides of a regular polygon increases, the measure of one of its angles increases as well. Students may make an intuitive answer here, using their visualization skills. As the number of sides increases, the angles have to open “wider” to accommodate the extra sides. The best way to see if the rate of change is constant is to make a table and check. See Part e.

c. 162°. A tessellation cannot be made with a 20-gon. As students found in class, the key to a tessellation is to have the angles around a point sum to 360°. Two 20-gons could be placed together, but that would only leave 36° to completely surround the point.

d. Students could look at their tables for the answer, but they should also work with the symbols in the formula. The numerator is 180n – 360. The denominator is n. This will only give whole number answers when n is a factor of 360.

e. Use \( y_1 = \frac{180(x - 2)}{x} \) to make a table as below.

<table>
<thead>
<tr>
<th>X</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>108</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>126.57</td>
</tr>
<tr>
<td>8</td>
<td>135</td>
</tr>
<tr>
<td>9</td>
<td>140</td>
</tr>
</tbody>
</table>

Students will then need to explain why the only regular polygons that will tile the plane are those with 3, 4, or 6 sides. They will need to use the ideas discussed in Part c.

In Courses 2–4, students investigate the behavior of rational functions like \( y = \frac{180(x-2)}{x} \) or \( y = 180 - \frac{360}{x} \). They will be able to visualize the function, describe the shape of the graph, and explain why it is always increasing, though at a decreasing rate.
Lesson 3, page 408, Modeling Task 1

a.

<table>
<thead>
<tr>
<th>Strip Pattern Symmetry Type</th>
<th>Mesa Verde</th>
<th>Begho</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Examples</td>
<td>Percentage of Total</td>
</tr>
<tr>
<td>Translation symmetry only</td>
<td>7</td>
<td>4%</td>
</tr>
<tr>
<td>Horizontal line symmetry</td>
<td>5</td>
<td>3%</td>
</tr>
<tr>
<td>Vertical line symmetry</td>
<td>12</td>
<td>7%</td>
</tr>
<tr>
<td>180° rotational symmetry</td>
<td>93</td>
<td>53%</td>
</tr>
<tr>
<td>Glide reflection symmetry</td>
<td>11</td>
<td>6%</td>
</tr>
<tr>
<td>Glide reflection and vertical line symmetry</td>
<td>27</td>
<td>16%</td>
</tr>
<tr>
<td>Both horizontal and vertical line symmetry</td>
<td>19</td>
<td>11%</td>
</tr>
<tr>
<td><strong>TOTALS</strong></td>
<td>174</td>
<td>100%</td>
</tr>
</tbody>
</table>

(These percentages have been rounded to the nearest whole number.)

b. The Mesa Verde prefer 180° rotational symmetry and Begho prefer horizontal and vertical line symmetry.

c. Pattern i shows vertical and horizontal line symmetry, whereas patterns ii and iii show 180° rotational symmetry.

i. Begho

ii. Mesa Verde

iii. Mesa Verde

*This kind of analysis of what is typical and how much variation there is, leading to conclusions about categorizing new discoveries, is useful in many different areas of research.*
Lesson 3, page 411, Organizing Task 3

a. Translation symmetry, 180° rotational symmetry around the origin, vertical line symmetry, and glide reflection symmetry

b. Translation symmetry and 180° rotational symmetry around the origin

c. No translational symmetry. But it does have 180° rotational symmetry about the origin.

d. No translational symmetry. This graph has vertical line symmetry only.