ISSUES in EDUCATION

College Mathematics Placement
Helping students make the transition from
Contemporary Mathematics in Context
to university-level mathematics

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Is everyone ready?
Both students and their parents want to know whether high school students using a research-based, integrated mathematics program such as Contemporary Mathematics in Context will be prepared for mathematics at the university level. They also want to know if universities are ready to teach high school graduates who have studied mathematics using such a contemporary, research-based curriculum. How will the changes in program emphases in this kind of high school mathematics program influence student placement in first-year college mathematics courses? How do universities determine first-year placement?

Located in the northwestern Detroit suburb of Bloomfield Hills, Michigan, Andover High School is one of 19 pilot schools in the Core-Plus Mathematics Project (CPMP), and we are now in our fifth year using the CPMP curriculum, Contemporary Mathematics in Context. Graduates of Andover High School, in nearly every case, plan to complete some form of post-secondary education. In the case of the graduating class of 1997, 219 of 220 graduates enrolled in a college, university, or technical school for the 1997–98 academic year. Approximately two-thirds of these students studied secondary mathematics in a curriculum based on Contemporary Mathematics in Context. Beginning with the graduating class of 1998, all Andover students will use Contemporary Mathematics in Context as the primary textbook series.

We have sent about 150 students on to college after high school study in a mathematics program based on the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989). Our work with college instructors and admissions personnel has raised several issues and questions. Students' and parents' anxieties about college mathematics placement have led to additional questions and concerns. These questions are very important to teachers who implement programs based on the NCTM Standards.

How have undergraduate mathematics programs changed?
The breadth of undergraduate mathematics programs has increased. Up until a decade ago, high school teachers could anticipate, with very few exceptions, the expectations and content of first-year college mathematics courses. Course titles such as Intermediate Algebra, College Algebra, and Introductory Calculus all carried familiar meanings ten years ago. We knew that it was very likely that students would be placed in a lecture setting, perhaps with one or two recitation sessions during each week. We knew that students were ready to use previously studied algorithms, with a focus on manipulating symbols rather than on learning concepts. Students
were expected to learn processes and facts, but they
gained little experience in applying those processes
and facts.

Some colleges and universities have implemented few,
if any, changes in their mathematics programs during
the last decade. Others have adopted a laissez-faire atti-
tude and allow individual instructors to implement the
approach deemed most appropriate for a particular
course. This attitude has led to disjointed curricula. For
example, in some cases, different sections of the same
course are taught in lecture or cooperative-learning for-
mat, with or without calculator use, with emphasis on
applications or algorithms, all within the same uni-
erity. Certain universities, such as the University of
Michigan and Duke University, have taken a more col-
laborative path: They have established common
philosophies for their calculus courses that emphasize
concept learning and application as well as communi-
cation and calculator technology to support investiga-
tion and understanding.

Secondary school mathematics curricula are also
more varied today. Many middle and high school pro-
grams now reflect the NCTM Standards, but many
do not. Students with broader sets of high school
learning experiences enter colleges with broader sets
of expectations and philosophies in beginning mathe-
ematics courses. The result: Placing these students into
the college program is now a more complex and chal-
lenging task.

**What can teachers and their students expect to find on mathematics placement exams?**

Last summer, university mathematics professors, lectur-
ers, and instructors met in Traverse City, Michigan, at a
conference sponsored by the Michigan Section of the
Mathematical Association of America. One session at
the conference addressed mathematics placement pro-
cedures. It was evident from the discussion that college
first-year mathematics placement is currently based
primarily on students’ knowledge of algebraic manipu-
lation and their knowledge of procedures. No college
represented at the conference evaluates student
ability in data analysis, probability, or discrete math-
ematics. Few colleges test understanding of geometric
concepts.

Placement procedures vary widely. Michigan State
University, for example, relies on a 28-question multi-
ple-choice test available to students on the World Wide
Web. (A sample is available at http://math.msu.edu.)
This assessment is supported by the ACT mathematics
subscore. The combination of placement-test score and
ACT score determines whether students will be
required to complete (a) a noncredit, remedial mathe-
matics course (0–9 correct), (b) a precalculus course
(10–19 correct), or (c) one of several calculus-level
courses (20 or more correct). The latter two options sat-
ify the university’s mathematics requirement. Students
scoring satisfactorily in a proctored setting may be able
to waive the university mathematics requirement altogether. MSU
allows students to use calculators on placement exami-
nations, and while most of the questions test knowledge of
symbolic algebra, some questions test knowledge of
geometric concepts, trigonometry, number sense, and
graphical representation of functions.

The university educators were asked, "If a high
school designed its mathematics curriculum based on your placement test,
would you be satisfied with that program?" The
unanimous response was, "No." Expect to
see placement procedures evolve in the coming
years.

While MSU relies heavily on the placement examina-
tion, the University of Michigan uses a somewhat differ-
ent method. It bases placement recommendations on a
combination of grade point average, college entrance
examination score, mathematics subscore (either ACT or
SAT), and achievement on a 25-question, 25-minute sym-

doic computation test. U. of M. draws questions from
materials available through the Mathematical
Association of America; most of the questions are tradi-
tional algebra questions. Students receive placement
advice, but they are free to enroll in any course they
choose.

None of the participants at the Traverse City conference
session expressed confidence that current placement
tests are adequate in assessing students who come from
contemporary, research-based mathematics programs,
yet there was a general understanding of the pressure
on high school teachers to prepare students for the
tests. The university educators were asked, "If a high
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How can teachers help students prepare
for entrance examinations and placement
tests?

Teachers cannot gear their instruction to a particular
university program or placement test under current
circumstances. Until colleges broaden the scope of their
placement examinations or begin to use other methods
for placement (such as requesting placement advice
from high school teachers), secondary teachers can
include learning activities in mathematics instruction
that will afford students the best opportunities for

(Placement, continued on page 4)
optimum college placement. The opportunities that already exist in Contemporary Mathematics in Context are worth highlighting. The activities include algorithmic/procedural supplementation, attention to the formal language of mathematics, and the development of a taxonomy, or classification, of mathematical terms and concepts.

**Maintenance**

Beginning in Unit 4 of Course 1, Contemporary Mathematics in Context includes maintenance supplements in the Teaching Resources. These tasks offer periodic review and additional practice in computational skills, procedural tasks, and algorithms. The work sheets present the content in the investigative-learning format and style on which Contemporary Mathematics in Context is based. Teachers may use these activities to support the content learning of the current lesson, or they may use the activities at other times to review or bring a slightly different perspective to topics that have been studied previously. The maintenance materials and other supplemental activities of the teacher's choice help students succeed in settings where mathematics problems are not contextualized.

At Andover, we use a mix of Contemporary Mathematics in Context and teacher-produced activities as homework. This combination offers two benefits. First, by not spending lengthy class time on tasks designed to build algorithmic skills, we prioritize “mathematics in context” above “mathematics as rote learning.” Additionally, when students have the more rote type of activity for homework, parents recognize more of the mathematics and are more likely to be able to help their children. This connection to more traditional mathematics curricula can help build community support for the integrated mathematics program.

**Technical language**

Success with mathematics activities may increase if we encourage students to learn the terms of mathematics and mathematical computation, such as “rationalizing” and “isolating the variable.” For example, a student may be aware that a quadratic function that crosses the x-axis at x = a and x = b has linear factors (x-a) and (x-b). If that student connects this concept with the terms “factoring” and “roots,” then he or she is able to communicate a conceptual understanding. I discovered, through a conversation with the father of one of my students, what happens when a student does not make this connection. The parent telephoned to express concern that his son was not learning the basics of mathematics. During our conversation, it became clear to me that the problem was not the student’s lack of understanding, but rather his inability to interpret the questions his father had posed. Contemporary Mathematics in Context provides numerous opportunities to introduce and reinforce the use of formal mathematical language. For example, investigation questions and the student materials that appear after a “Checkpoint” section in many lessons often include definitions of highlighted mathematical terms. In addition, students should regularly be encouraged to use appropriate mathematical terminology and symbols in their class notes and journals.

**Taxonomy of mathematics**

Technical language extends to include broad terms such as “algebra,” “geometry,” and “statistics.” Students need to clearly understand the definitions of these broad terms in order to communicate effectively about their learning. We should carefully assist students in classifying the terms, procedures, algorithms, concepts, and problem types according to related mathematics strands. In Course 1, Part A, Contemporary Mathematics in Context students begin to construct Math Toolkits in which they record and organize mathematics concepts and methods by strand. Students continue to add to and use this set of tools throughout their study of mathematics. Word-processing technology and other software, such as Microsoft’s Research Organizer, can help students to revise, organize, and present the Math Toolkit information.

**What is most important for our students’ success?**

I offer one final note about preparing our students to be appropriately placed in their first college mathematics courses. Professor B.A. Taylor, Mathematics Department Chair at the University of Michigan, wrote:

> At the national level, it will probably take some time for universities’ placement procedures to adjust to the varying curricula in use nationwide. However, at Michigan we are not too concerned with the particular curriculum students have followed. We are concerned with the level of mathematical skills and sophistication they have developed. In fact, the range of backgrounds for students using identical curricula but from different schools is at least as wide as that of students from different curricula.

This is a time of emotion and anxiety concerning mathematics education. Parents, teachers, counselors, administrators, and peers all have the power to influence a student’s confidence about the mathematics under study. As teachers, we must do all we can to minimize the anxiety levels of our students. There is a great ongoing debate, but we do not need to bring our students into the center of this debate. Let’s all become advocates of change regarding the placement procedures of the colleges and do all that is in our power to highlight for our students the positive aspects of contemporary, research-based mathematics curricula. It is in their best interest, and ours, that we do this.
Given the opportunity, what would you ask the authors of the mathematics program you teach? Here’s your chance! In Ask the Authors, program authors respond to teachers’ questions about curriculum, instruction, assessment, and program content. You may submit your questions using the Suggestion Box form on page 15.

**Q:** Teachers in my school are really excited about teaching Contemporary Precalculus through Applications. They are learning lots of new mathematics and the students are becoming good problem solvers. But, I’m the Advanced Placement Calculus teacher, and when I have these students in my class, they often cannot do algebra. In the old days, my students could easily simplify the derivative of \( f(x) = x(x-6)^{2/3} \) to find the relative extreme. Now, they act befuddled. What is mathematics coming to?

**A:** First of all, my students cannot do that problem either. But, if I remember correctly, I have never had students who could do that problem very well. The difference is that now we have to take the time in calculus to teach students the harder algebra ideas as they need them. My students enjoy this challenge as a change in routine, and now that I am planning for it, the time I spend on these topics is better organized and more efficient.

In particular, the solid background of my students in functions, graphing, algorithms, and problem solving allows me to spend time on algebra when I need it. I have to change how and what I teach my calculus students because of their background knowledge from Contemporary Precalculus through Applications. In fact, my students are challenging me to rethink the goals and emphases of the entire calculus curriculum. I think my course and my students will be better as a result—and so will I.

Dr. Jo Ann Lutz, one of the authors of Contemporary Precalculus through Applications and Contemporary Calculus through Applications, answered the above question. This particular question and answer appeared previously in the “Dr. Discrete” column in a mathematics newsletter published by The North Carolina School of Science and Mathematics, where Dr. Lutz is a faculty member.