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ISSUES in EDUCATION

Collaborative Groups Promote Active Mathematics Learning

by Jacqueline Stewart; Teacher; Contemporary Mathematics in Context; Okemos High School; Okemos, Michigan

What can be gained by organizing part or all of your mathematics instruction around the collaborative-groups model?

Traditionally, mathematics teachers have instructed by explaining algorithms, theorems, and sample problems to large groups of students who are presumably interested in the topic at hand. Lectures may be an efficient means of transmitting information and facts from one mind to another. However, lectures may not effectively create opportunities for students to make the mathematical thought their own or allow teachers to evaluate understanding. By comparison, collaborative group work requires students to verbalize their ideas, hear the ideas of their peers, and challenge one another. The main benefit from instituting group work, therefore, is that within that framework students have a chance to think for themselves and share their developing thoughts. This process could be as simple as a teacher posing a series of questions about the properties of polygons that students discuss in small groups and report on in a large group discussion. Alternatively, groups might be involved in a more elaborate investigation that guides students in setting key criteria for classifying plane shapes. In these exchanges, teachers can more easily assess student progress and identify individual strengths and weaknesses.

Two examples from my classroom

In an early experience with group work, my ninth grade students set up experiments to determine the relationship between the weight of a "bungee" (actually fishing weights) and the length of the rubber band that supported the weight. Each person had an assigned role, such as measurer, recorder of data, or grapher. At the end of the experiment, each group was asked to prepare a summary and make a prediction from the results. Individuals from groups summarized what their groups found so that the whole class could look for similarities and differences. Each group found a pattern that allowed group members to make predictions, but it was not until the whole class reassembled that students could see the overall picture: All the patterns were linear and students were using a constant

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Ninth grade CMIC *students explore the relationship between the weight of an object and the length of a rubber band supporting it.*

(Collaborative, continued from page 1)

rate to make predictions. In this way, each person in the small group contributed to the group's discovery of a pattern, and each group contributed to the class's overall idea of linearity. Roles and products were clearly defined.

In another example, older students discussed and conjectured about the properties of special quadrilaterals. Once they had listed conjectures and appropriate assumptions, I assigned each group one proof. Groups wrote their proofs on sheets of newsprint, explained them to the class, and posted them on the bulletin board. More than one group had the same assignment, so there was some diversity in the proofs that students constructed. This time I assigned no specific roles because these students were accustomed to group work and contributed responsibly to the group product.

I could have made this an individual assignment, but students often find proofs difficult. They do better when they have to verbalize their reasoning before committing it to paper. In the long run, each group did three or four good proofs, instead of everyone doing 10 not-so-well-done proofs. The diversity in the methods was another plus, as was the sense of ownership in students having their "property" posted on the official board.

What are the drawbacks in instituting collaborative group work and how can these drawbacks be overcome?

There can be drawbacks to using the collaborative-groups instructional mode. Some of the concerns center on accountability and fairness. Parents often fear that a few students will do all the work and others will slide by without contributing to the group. These concerns are fairly easy to address. Perhaps the most obvious answer to parental concerns is that a student cannot be less successful as an individual simply because someone else in the group is not performing responsibly.

In addition to communicating with parents, teachers can rotate group membership frequently, make observations about group members' contributions, talk to nonproductive students, make everyone in the group responsible for turning in individual work, and give grades for group work. For example, a teacher might assign each member of a group to explore two cases for the equation $y = a + bx$ and report and explain his or her conclusions to the group.

When teachers think about group work, they tend to be concerned about how to create or find problems that require students to collaborate and communicate and how to provide a structure for learning. For example, a teacher may look for a problem that asks a group of students to conduct an experiment with angle measurement, answer questions about their findings, and give evidence to support their claims. Teachers also have concerns about motivating unmotivated groups, dealing with disruptive students, and keeping track of progress within groups. Teachers resolve these concerns by clearly defining group work, selecting appropriate and engaging content for group work, setting criteria for quality group interaction, and carefully monitoring group work.

The characteristics of group work

Group work creates opportunities for students to make sense for themselves of an important mathematical idea, such as patterns of periodic change. Ideally, group work brings students into contact with a mathematical idea in an attractive, puzzling, or interesting problem-oriented setting and lets them interpret and make their own sense of the idea, without the teacher's "educated" mind doing the translation for them. For example, students studying the patterns of periodic change work in groups to identify and describe real situations that involve periodic change, sketch the pattern of change and then compare their situations and sketches with those of other groups. Later in the lesson, students progress to describing periodic graphs in terms of amplitude and period. Students can count on other students to contribute their interpretations and ideas in the group setting. Communication is necessary to reach agreement. The teacher, meanwhile, actively participates in group work: probing, redirecting, and orchestrating large-group discussions and summaries.

Group work is productive. Students might submit a list of various methods of finding area and perimeter, a formula for solving a ratio problem, or a graph and summary of a group's experimental data. The product should reflect the input of the group and not just any one person. These various sources are often apparent in the products of group work, including a "minority report" from a dissenter, a summary of individual strategies the group has tried and rejected, or the expression of a dual perspective. Collaborative group work means that each mind is engaged in a worthwhile problem, using the typical resources of teacher and text, but also extending to other students' interpretations of the problem. If the work is divided among individuals, each person should add an important piece of information to a common discussion of a larger problem.

To describe what group work is, it might be necessary to describe what it *is not.* Group work is not dividing up a repetitive task into small pieces, such as assigning five group members 10 algebra equations each from a page of 50 problems. It is not separating a set amount of work into a number of unrelated tasks that are done by individual members of the group and then turned in for a combined grade. It is not lacking in structure or accountability. And it is not appropriate for every lesson or for every group of students.

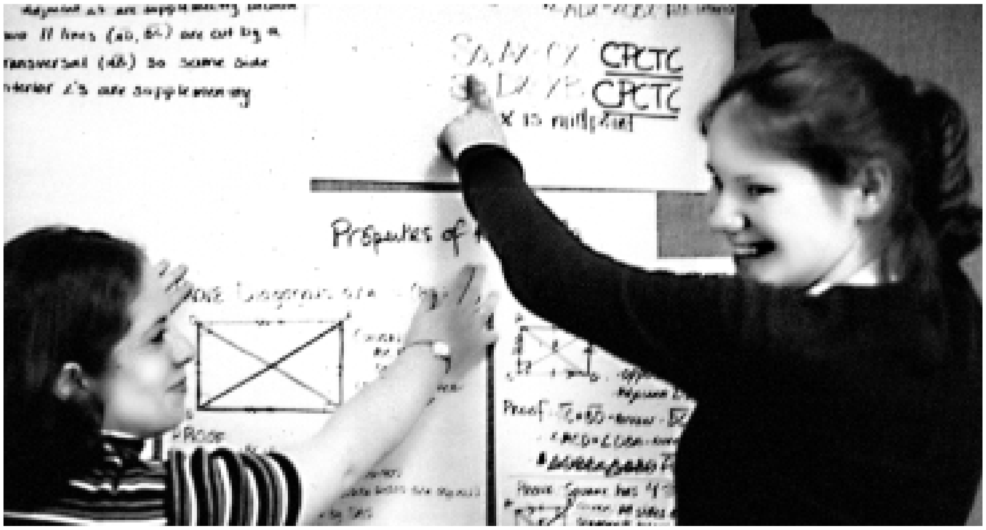
"So how do I get started?"
PRACTICAL TIPS

Gather appropriate problems and tailor them to meet your students' needs.

First, you need mathematical problem situations that are appropriate for group work. If the problem can be done just as well by an individual, it will probably not allow students to reap the benefits of group work.

(Collaborative, continued on page 4)

Group work is . . .	Group work is not . . .
• A set of tasks that allow students to make sense for themselves of an important mathematical idea.	• Dividing up a repetitive task into small pieces.
• A way to bring students in contact with a mathematical idea in a problem-oriented setting.	• Separating a set amount of work into a number of unrelated tasks, which are then completed by individual group members for a combined grade.
• Productive: Students solving problems, clarifying ideas, and creating examples that reflect group input.	• Lacking in structure or accountability.
• Activity that engages each mind in a worthwhile problem.	• Appropriate for every lesson or every group of students.



After group work on proofs, students explain their reasoning to the whole class.

(Collaborative, continued from page 3)

Similarly, if you find yourself telling each group how to proceed with a solution (deriving the Law of Cosines, for example), then it might be better to do the lesson as a large group. *Contemporary Mathematics in Context (CMIC)* can be used for small-group discussions exclusively or as the blueprint for discussions in a combination of large- and small- group settings. Teachers use *CMIC* in a variety of group situations. For example, in planning for a lesson, I read over a structured "Investigation" carefully and decide if my students will need some help to get started or if I will give a brief motivating introduction. I consider how many activities my students will be able to complete before the whole group summarizes discoveries or synthesizes information.

I ask myself some questions: How long will this take? What are some possible pitfalls, and what can I do to smooth the way without eliminating productive struggling? How should groups be held accountable for their work—by demonstrating solutions or by leading a summary discussion? Do I want one group or all groups to report? Will a group lead the "Checkpoint" discussion, or should I lead it? Will the class end with students turning in their written "Checkpoint" summaries as part of a notebook grade, or should I assign an "On Your Own" example to hold each person responsible for mastering today's lesson?

Define roles and form groups.

I have found that it is best to define the roles for each student for the class's first experiment with collaborative group work. *CMIC* suggests some roles for group members, including coordinator, measurement specialist, recorder, and quality controller. Make sure to communicate your expectations for products or outcomes. Later, your students can be more flexible about contributing and the problems can be more open-ended, with fewer rules from you about the nature of the end product.

You will also need to decide how to form groups. Usually, I favor random groupings because even though I think I know what is best for my students, my biases may cause me to misjudge potential groupings. If random groups are created and you see obvious problems, you can subtly make alterations. Sometimes I plan heterogeneous groups deliberately (assuming I am all-knowing enough to accurately judge individual students' ability levels!) and sometimes I will take attributes such as gender, work ethic, and reading and writing abilities into consideration. Groups of three or four seem to work well. You will be able to spend some time with each group as you circulate among them. Just do it—and get past worrying about the mechanics!

Monitor groups and model collaborative behavior.

Now that you have formed the groups and assigned investigation materials, take a short time to talk with your students about what group work is and what it is not and why you want students to try group work. Then let them loose on the problems. If your students are not accustomed to collaborative learning, they may be awkward at first. Arm yourself with a clipboard and a list of the names of the students in each group, and promise yourself you will not resume the "sage" role until the problem has been solved, or until two or three days have elapsed, whichever comes first, even if you think things are not going very well. On the clipboard, write down both obvious problems and examples of cooperation. If you hear students asking each other for clarification, try to write down the exact words they are using. If you hear unhelpful comments, do likewise. If you observe some students listening well, or others interrupting, capture the moment as faithfully as you can in writing. If some group members persist in ignoring one another and petitioning you for help, try to deflect this behavior by answering questions with a hint and another question, rather than a full explanation.

When you sit with a group, try to stay there for 10 minutes, long enough for you to get a feel for how the work is going and to model some collaborative behavior with phrases such as, "So what I think you are saying is ..." or "I don't quite get your explanation; can you give me an example, or explain it another way?" or "I wonder if your method would always work—for example ..." These questions help students make sense of the mathematics without superimposing too much of your own understanding too early. It takes practice to frame questions in ways that help students move forward; it seems much easier just to tell what you already know. There is a place for telling, but it comes after you have determined that students have done their best and cannot be further redirected with a hint or a question.

The more you can involve your students in the process of setting the targets and standards for good group work or reports, the better they will understand how they have earned their grades.

Facilitate group reporting and share observations.

At the end of the first attempt at a group investigation, have each group report on its progress on the problem, focusing primarily on the mathematics involved. Then change the focus of the discussion to the dynamics of the group work, and ask students to state what worked well and what did not. As part of this discussion, tell your students you have been keeping notes on what you observed and then share some examples—without sharing names; individuals will recognize themselves! I usually try to make general comments about how a group is working and then mention details that show that I heard each person making a valuable contribution (even if it is sometimes hard to catch students making meaningful statements.) These group observations can thus be part of an informal check on an individual student's progress. Students will see that there is a plan in place for sharing the products of group work. Giving each group some feedback, either on a group evaluation form that has been mutually accepted or giving feedback on oral summaries or just offering a few words about what you observed that did go well— and what did not— gives students some guidance about what you would like to see in the future. Ask the class to help you decide on the criteria to use for evaluating group work. This request will elicit ideas for a beginning framework that you can refine later.

Grading

I assign a small percentage of a student's overall grade to group work. Students feel accountable for being a productive group member, but parents don't receive the impression that a student's overall grade depends a great deal on the student's interactions with others. This group grade can be determined by using the set of criteria your class helps you build to evaluate the quality of group interaction. You may also use those criteria in conjunction with a group grade for a product such as a summary report. The more you can involve your students in the process of setting the targets and standards for good group work or reports, the better they will understand how they have earned their grades.

Rewards of successful collaborative group work

When group work is going well, the atmosphere in the classroom is positive, buoyant, and welcoming. Visitors notice that the noise level is higher than it is in a lecture setting, but not unusually high. And they have difficulty figuring out what the teacher is doing to keep things under control. One visitor to my classroom remarked, "But you sat down with your back to most of the class!"

In fact, an interesting investigation and the opportunity to work on it with supportive colleagues drives positive student behavior. Students look forward to oral presentations as a chance to show their prowess and to receive comments from other groups and from me. On successful days, I enjoy being part of a community of learners. My clarifications are valued because they are needed, and my perspective is appreciated because I know the road ahead. Unexpected insights, odd digressions, and challenging questions that I really may not be able to answer right away keep me interested and motivated too. But the ultimate reward is seeing students increase their curiosity and gain confidence and knowledge.

Jacqueline Stewart teaches Contemporary Mathematics in Context *Course 3 and Course 4 this school year. She has been a teacher for 23 years (in three countries!) and has been teaching* Contemporary Mathematics in Context *for five years.*

If you are interested in reading more about collaborative group work, turn to the review of *How to Use Cooperative Learning in the Mathematics Class* in "Mathematics Bookshelf" on page 9 of this issue.

For Your Information

Cooperative group work, collaborative group work, *or simply,* group work *are terms often used interchangeably in professional literature for teachers. The Core-Plus Mathematics Project authors have chosen most often to use the descriptor "collaborative" to convey the idea that the group work focuses on joint intellectual or research endeavors. In most collaborative learning activities in* Contemporary Mathematics in Context, *small groups of students work jointly on a problem or task rather than subdividing the task and then sharing and piecing together a final solution.*

Working with ESL Students in the Integrated Mathematics Classroom

by Kathy Harris; Teacher; Contemporary Mathematics in Context; *Vallivue High School; Caldwell, Idaho*

tip

Vallivue is a rural school district with a population of about 900 students in grades 9–12. The largest minority group (15–20 percent) is Hispanic. Most of the English as a Second Language (ESL) students are from Mexico and speak Spanish; a few ESL students are Vietnamese or Chinese speakers.

I have found that ESL students often have great success in the *Contemporary Mathematics in Context (CMIC)* classroom. In fact, they tend to do better there than they do in traditional classes. I believe that the cooperation and discussion of mathematics in groups (in any language) help students to build a better foundation upon which they can continue their study of mathematics.

Harold L. Schoen, Core-Plus Mathematics Project (CPMP) Evaluation Director, spent some time in one of my *CMIC* classes and interviewed several ESL students along with our district ESL coordinator. The following comments are from a transcript of the interview with the ESL coordinator:

This is the first class [Course 1] where I see that "John" [an ESL student] really had to learn to think; he has now had a chance to show what he knows and I see that reflected in his other work as well. Now he cares about school whereas before he didn't. He now has the opportunity to say, "Listen, I am smart, I do have something, and I can show what I can do."

I have found that there are several actions that I can take to help ESL students succeed in mathematics. The following three suggestions may help you better teach ESL students in a *CMIC* classroom:

1. Work with the ESL teacher for proper placement of ESL students.

I have worked closely with the ESL coordinator in our district to help ensure that our students who have limited English are still allowed the opportunity to learn mathematics. We make sure that each ESL student is placed in an appropriate mathematics class. We take into consideration the educational background of the student along with his or her age and maturity level. Limited English does not mean limited mathematics skills. We have successfully placed students in *CMIC* Courses 1, 2, and 3 without requiring earlier courses as prerequisites.

In addition to placing students in mathematics classes at appropriate levels, we also try to place at least two same-language ESL students in a given classroom. This allows the students to collaborate and discuss the investigations with each other in their native language. Communication is such an important part of mathematics (and is so thoroughly built into the *CMIC* curriculum) that it is essential that students have the opportunity to practice this skill.

2. Involve classroom translators in your instruction.

In my school, every classroom teacher who has an ESL student is assigned a classroom translator. We use both adults and older students as translators. A classroom translator will work with two to six ESL students in a class. Adult educational aides are assigned to work in classrooms that have large groups of ESL students, students who have no English skills (recent immigrants), or teachers who have no experience with ESL students. Bilingual students who work as translators must be juniors or seniors who have passed Course 3, Algebra II, or a higher-level mathematics class. The student translators receive high school credit for a class entitled Advanced Communication. All translators work closely with the classroom teacher and the ESL coordinator.

3. Require participation from all students.

All students are required to participate in all activities. I make accommodations for ESL students so that they can be successful in the activities. Translators help with reading and discussion in groups. I allow the students to give their presentations in their native languages and then have the presentations translated for the rest of the class. Depending on the English ability of the students, I may accept written work in Spanish. Tests are read orally and translated for students with limited English, and extra time is allowed for some homework assignments. These accommodations allow the ESL students to participate and learn as much mathematics as possible.

tip trade

is the place to exchange hints for effective instruction and classroom management. Share details about activities, timesavers, extensions, and other creative ideas you use in your classroom. Use the *Suggestion Box* form on page 15 to submit your ideas.

from the Author's

COURSE 3 *CONTEMPORARY MATHEMATICS IN CONTEXT*

by Christian Hirsch, Director,
Core-Plus Mathematics Project

Following three years of research, development, and evaluation, *Contemporary Mathematics in Context (CMIC),* Course 3 is in the process of being published. Part A is available now. Like the two previous courses, Course 3 features four interwoven strands: algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. The course is composed of seven units and a thematic capstone. A capstone is an extended project-oriented activity that enables students to apply important modeling concepts and methods developed throughout the course.

CMIC Course 3 Units

Unit 1	*Multiple-Variable Models*
Unit 2	*Modeling Public Opinion*
Unit 3	*Symbol Sense and Algebraic Reasoning*
Unit 4	*Shapes and Geometric Reasoning*
Unit 5	*Patterns in Variation*
Unit 6	*Families of Functions*
Unit 7	*Discrete Models of Change*
Capstone	*Making the Best of It: Optimal Forms and Strategies*

A major focus of Course 3 is the formalization of mathematical ideas, many of which were developed by students in previous Core-Plus courses as they solved mathematical problems in contemporary contexts. Formal and symbolic reasoning strategies, the hallmarks of advanced mathematics, are developed here as complements to more intuitive arguments and numerical and graphic approaches to problems.

As students work through Unit 1, *Multiple-Variable Models,* they are confronted with situations modeled with complex rules and relationships involving three or four variables and requiring the need for additional symbol manipulation. Reasoning strategies students have previously used informally as they simplified expressions, solved equations, and operated with algebraic symbols are connected to number system properties in Unit 3, *Symbol Sense and Algebraic Reasoning.* The notion and notation of function are formalized, and proof through algebraic reasoning is introduced. Because of the strong conceptual development in Courses 1 and 2 and the continued attention to developing mathematics in context, the algebraic symbol manipulation (including work with factoring and the quadratic formula) makes sense to students.

In *Families of Functions,* students summarize their toolkit of functions, the patterns of variations modeled by each type of function, and the representations of these patterns in numerical, graphic, and symbolic forms. Techniques are developed for customizing basic function rules to model data patterns whose graphs are transformations of familiar forms. This toolkit of function understandings and skills provides strong preparation for students as they progress to applications of mathematics in other disciplines and as they continue their mathematical studies leading to calculus.

Blending ideas from algebra and discrete mathematics, Unit 7 formalizes the development of recursion, a major theme of the *CMIC* curriculum. The familiar *NOW-NEXT* thinking first developed in Course 1 is analyzed more carefully through combined recursion equations of the form $A_n = rA_{n-1} + b$. This approach unifies topics often seen as disjointed. For example, arithmetic sequences (and related series) arise when $r = 1$. The case when $b = 0$ leads to familiar geometric sequences (and related series).

Formal reasoning in geometric contexts and formal organization of geometric facts and properties are the primary foci of Unit 4. Again, connections among strands of the curriculum are highlighted and used to efficiently develop important geometric ideas. For example, in Unit 1, the Law of Sines and the Law of Cosines are examined as multiple-variable models; these laws are proved using algebraic reasoning in Unit 3 and then are used in Unit 4 to derive all the familiar triangle similarity theorems. Students easily and efficiently prove the congruence theorems for triangles by applying corresponding similarity theorems when the scale factor is 1. Experiences with synthetic and coordinate approaches to proof in this unit further underscore the connectedness of mathematics and encourage flexibility in approach.

Course 3 is the final year of this three-year NCTM *Standards*-based program intended for all students. As such, like the two previous courses, Course 3 seeks to prepare students not only for success in college, but for success in careers and in daily life. These multiple goals are reflected in the choice of content, which includes mathematics broadly useful in business and industry. Overall, the curriculum emphasizes

(Contemporary, continued on page 11)

research & evaluation

RESULTS FROM THE THREE-YEAR NATIONAL FIELD TEST OF THE *CONTEMPORARY MATHEMATICS IN CONTEXT* CURRICULUM

by Harold L. Schoen, Evaluation Director, Core-Plus Mathematics Project

Before adopting a new curriculum, a school district should examine available evidence that the curriculum will enable students to learn important mathematics at least as well as available alternatives. To provide interested schools with such data, each *Contemporary Mathematics in Context* (*CMIC*) course is field-tested for a full school year following two years of careful development. This national field test is conducted in 36 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. A broad cross-section of students from urban, suburban, and rural communities with ethnic and cultural diversity is represented. Course 1 was field tested in 1994–95, Course 2 was field-tested in 1995–96, and Course 3 was field-tested in 1996–97. The field test of Course 4 will be conducted in 1998–99. Some of the main results from the field test of the first three courses are summarized below.

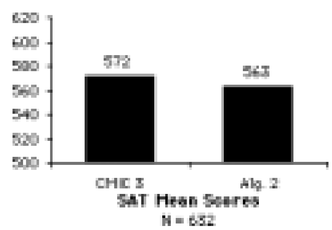
***CMIC* students perform better than non-*CMIC* students on mathematics achievement tests.**

- *CMIC* students significantly outperformed non-*CMIC* each year on the mathematics subtest of the nationally standardized Iowa Tests of Educational Development (ITED). This finding was consistent for students with a wide range of entering mathematical achievement including those in the top quartile, for boys and girls, for students of all ethnic backgrounds, and for students in rural, urban, and suburban schools. In Courses 1, 2, and 3, *CMIC* students' average performance was significantly better than that of the nationally representative norm group. In Courses 1 and 2, *CMIC* students' average performance was significantly better than that of the traditional comparison groups in some of the same schools. There was no traditional comparison group in Course 3 since too few of these students remained in mathematics courses and were available for testing.

- On a Core-Plus project post-test given at the end of Course 2, *CMIC* students illustrated significantly better understanding than non-*CMIC* students of algebraic and geometric concepts that both groups had the opportunity to learn. They were also better able to reason with and apply methods of algebra and geometry. There was no difference in mean performance between *CMIC* and non-*CMIC* students on a subtest of basic algebraic procedures.

- On a test consisting of released 12th grade items from the 1990 and 1992 administration of the National Assessment of Educational Progress (NAEP), *CMIC* students' mean performance was considerably higher than that of a nationally representative sample of 12th grade students in all of the following areas: statistics and probability, measurement, algebra and functions, geometry, numbers and operations, conceptual understanding, problem solving, and procedural skill.

- *CMIC* students do as well or better than comparable non-*CMIC* students on the SAT and ACT college entrance exams. For example, the SAT mathematics mean scores for *CMIC* Course 3 students and for Algebra II students in 10 field-test schools are shown below.



***CMIC* students have a more positive perception of their developing mathematical talent than non-*CMIC* students.**

- On a written survey administered at the end of Course 2, *CMIC* students were significantly more positive than non-*CMIC* students about their conceptual understanding, problem-solving ability, mathematical reasoning ability, ability to communicate mathematical ideas, and interest in mathematics.

- This pattern of perceptions was evident for students at all levels of mathematical achievement.

Overall, the field-test evidence shows that *CMIC* students develop better conceptual understanding and problem-solving skills, have more positive attitudes about mathematics than students in traditional curricula, and do at least as well on college entrance examinations.

mathematics bookshelf

Book Review

How to Use Cooperative Learning in the Mathematics Class
by Alice F. Artzt and Claire M. Newman
Reston, VA: National Council of Teacher of Mathematics
1997, Second Edition
81 pages

Reviewed by Christine Longcore

If you have ever wanted to introduce or enhance cooperative learning activities in your classroom, this book from the National Council of Teachers of Mathematics (NCTM) will serve as a valuable resource. Alice F. Artzt and Claire M. Newman, both from Queens College of the City University of New York, provide thoroughly researched, practical guidance that is widely applicable in both elementary and secondary classrooms. Artzt and Newman address both the underlying philosophy and the concrete practices that support productive cooperative learning experiences.

Artzt and Newman devote the first section of their book to defining cooperative learning, describing its benefits, and explaining specific ways to initiate and facilitate group mathematics activities. They attend to the vital questions teachers raise about cooperative learning, such as how to form groups and design tasks for group work. Three suggested ways to form groups are to mix students heterogeneously according to varying abilities or other characteristics, randomly assign students to groups, or allow students to select their own groups. According to the authors, mathematics problems for group work should require group members to depend on one another to complete the assigned task. Artzt and Newman also offer ideas on how to assign students to groups, how frequently to change group membership, how to decide how many students should be in each group, and how to create team spirit. The authors also address questions about assessing verbal communication, collaborative skills, and mathematical understanding on an ongoing basis during group activities. One of the most helpful aspects of this discussion is that the authors describe specific student behaviors that are part of successful group interactions. For example, students who are working successfully as a group "eagerly check with one another to be sure that each person understands the material, agrees with the results of conclusion, and can represent the group as a spokesperson." By reading descriptions of positive student behavior, teachers can learn to recognize signs of successful cooperative learning. The authors describe various options for cooperative groups. They suggest that students work together on developmental lessons in which students apply and practice newly learned concepts or participate in guided discovery. Other times that students may benefit from cooperative group learning include test-review and enrichment lessons. For example, students can complete a sample test for homework and then meet in groups to discuss the solutions in preparation for a test or investigate historical topics related to mathematics as an enrichment project.

The good news for secondary mathematics teachers is that this book contains eight sample activities for secondary students. (The book suggests a total of 15 activities. The first seven activities are for students in elementary through junior high school, so teachers may want to share this book with colleagues teaching lower grade levels.) Each activity includes a sheet of teaching notes along with a problem sheet and a record sheet for students. In one sample activity, groups of three students each graph specified parabolas by translating the graphs of other parabolas.

Secondary mathematics teachers will be able to use the front section for ideas to enhance the cooperative learning experiences using their regular curriculum materials. They can refer to the second section periodically to find appropriate activities to add to their lesson plans. For some teachers, this book may be the beginning of further investigation of using cooperative learning in the mathematics class. The bibliography will entice the curious to read more about research and practice in cooperative learning.

Secondary teachers will find numerous opportunities to put the authors' suggestions to use in the classroom. The strength of this book lies in the balanced mix of supporting research, well-structured sample activities, and practical advice for creating, facilitating, and assessing mathematics groups.

For more ideas about how to bring cooperative learning into your mathematics class, see Jacqueline Stewart's article in this issue, entitled "Collaborative Groups Promote Active Mathematics Learning" on pages 1–5.

mathematics bookshelf

Have you read any good books about mathematics lately? In Mathematics Bookshelf, tell other teachers about what you've been reading. We welcome book and article reviews about mathematical concepts, current research in mathematics education, and first-hand experiences of mathematics teachers. Use the Suggestion Box form on page 15 to tell us which book or article you would like to review.

ASK the Authors

Achieving Contemporary Goals in Mathematics Education

by Eric Hart, Ph.D., Senior Curriculum Director, Core-Plus Mathematics Project

From time to time, as *Contemporary Mathematics in Context (CMIC)* teachers and administrators you may be asked to explain how the curriculum achieves current goals in mathematics education. Following are six important points and some suggested responses that describe how *CMIC* meets specific objectives.

1 All students must learn basic skills.
More basic skills are taught in the CMIC *curriculum than have commonly been taught in the past.*

- Students learn not only basic skills in algebra and geometry, but also important skills in probability, statistics, and discrete mathematics.
- Students learn basic skills based on a foundation of conceptual understanding, and they apply those skills to solve significant problems.
- Students learn basic skills at the right time. For example, rather than introduce a heavy treatment of algebraic skills early, *CMIC* introduces these skills gradually in Course 1 and develops them more fully in later courses. This gradual development of skills works because it builds on the conceptual understanding and mathematical maturity that students develop as they progress through high school.
- Students learn essential basic skills, which have been included in the curriculum based on consultation with experienced teachers and a distinguished advisory board. Some skills have been added because they are essential in contemporary life, for example, skills in statistics. Other, obsolete skills have been deleted because they are no longer essential, for example, simplifying complicated rational and radical expressions.

2 Technology is a powerful aid to education, but it must be used wisely.
The CMIC *curriculum uses technology as a tool to promote thinking, never as a crutch to replace thinking.*

- Graphics calculators, more aptly called hand-held computers, are used judiciously as tools to promote conceptual understanding, problem-solving ability, and skills development.
- A general principle for the use of technology in the *CMIC* curriculum is that students first develop conceptual understanding and then they use technology to extend and deepen their understanding and to solve more complex problems.
- The use of technology permits the curriculum and instruction to emphasize multiple representations (numerical, graphical, and symbolic) and to focus on goals for which mathematical thinking is central.

3 We must regain international competitiveness in mathematics education. Countries around the world use an integrated mathematics curriculum.
The CMIC *curriculum integrates the four strands of high school mathematics—algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics.*

- A report analyzing the dismal below-average performance of the United States on the Third International Mathematics and Science Study (TIMSS) states:"How mathematics is arranged in courses [in the United States] also seems to be part of the problem. ... United States mathematics students take separate courses in geometry, precalculus, etc. In most TIMSS countries, students take a course in [integrated] mathematics—a course which may include parts of advanced algebra, geometry, finite mathematics, and calculus at the same time." (United States TIMSS National Research Center, 1998, p. 5)
- In the *CMIC* integrated curriculum, problem solving, conceptual understanding, and basic skills are all developed in the context of real-world applications that naturally cut across the different strands of mathematics.

4 All students should learn mathematics. A good mathematics curriculum must be both challenging and accessible, thereby providing a solid mathematics education for all students.
The CMIC *curriculum makes more mathematics accessible to more students and at the same time challenges even the most able students.*

- The *CMIC* curriculum is designed to provide a solid mathematics education for work-prep, tech-prep, and college-prep students alike. Differences in student background, performance, and interest are accommodated by the depth to which investigations may be pursued, by the rich variety of homework problems, and by the nature and complexity of applications.

- The *CMIC* curriculum has been used successfully with students ranging from remedial students who have never (yet) been successful in mathematics to top-scoring students in math and science magnet schools.

❺ **Algebra is essential.**
The CMIC *curriculum teaches more algebra to more students.*

- Algebra is developed systematically and rigorously in the *CMIC* curriculum. There are three fundamental ways to represent algebraic relationships: tables, graphs, and equations. *CMIC* students develop the ability to model and solve problems using all three of these representations.

- In Course 1, there is more emphasis on using tables and graphs to solve problems and develop conceptual understanding, with less emphasis on analysis of equations and other algebraic expressions. Based on the solid conceptual understanding that is developed in Course 1 and that continues throughout the higher courses, students do more extensive and rigorous algebraic work in Courses 2–4. Completing the development of all algebraic skills needed as preparation for calculus is given special attention in Course 4.

❻ **High school mathematics education must provide a solid preparation for college.**
The CMIC *curriculum prepares students to pursue college majors in mathematics and science as well as in any nontechnical field.*

- The *CMIC* curriculum provides the rigorous content needed for students to succeed in any first-year college mathematics course, including calculus and statistics.

- While still in high school, students are prepared to take Advanced Placement (AP) Statistics after three years of the *CMIC* curriculum, and they are prepared to take AP Calculus after four years.

- Students who have completed the entire published version of the *CMIC* curriculum will not reach college campuses until fall 2000. Even so, field-test evaluation results show that (a) *CMIC* students do as well or better than non-*CMIC* students on the SAT and ACT college entrance examinations; (b) *CMIC* students significantly outperform non-*CMIC* students on the mathematics subtest of the Iowa Test of Educational Development, which correlates even more strongly with success in college than the ACT and SAT tests do; and (c) students finishing three to four years of the field-test curriculum have been accepted at top universities around the country, including Harvard University, Boston College, the University of Michigan, the University of Chicago, Morehouse College, the Air Force Academy, the University of Arizona, and many others.

(Contemporary, continued from page 7)

mathematical modeling, intelligent use of technology, collaborative team work, and communication. Unit 1, for example, introduces students to the important tool of linear programming, a technique that reportedly accounts for over 50 percent of all computing time used for management decisions in business.

Mathematics for work is also a theme of Unit 5, *Patterns in Variation,* which provides students with experience in thinking about and working with the variation in data that results from measurements. When samples of measurements taken over time come from a distribution that is approximately normal, it is possible to establish a set of criteria involving the standard deviation for defining when a process is "in control." In this unit, students study and apply the mathematics underlying statistical process control, a method increasingly used in American industry to improve the quality of products.

Because public opinion influences education, politics, fashion, television, and many other trends in contemporary society, in Unit 2 students learn how to measure and interpret public opinion through voting and surveys and how to critically analyze surveys reported in the media. Understanding of sampling distributions and confidence intervals, fundamental to hypothesis testing in experiment-based settings, is also developed. Strategically placed to coincide with fall elections, this unit enables students to develop skills for effective citizenship and wise consumerism.

With Course 3 completed, the Core-Plus Mathematics Project has turned its attention to the development and evaluation of a flexible fourth-year course for college-bound students. An overview of Course 4 will appear in a future issue of *ELC's Mathlink.*

techLINK

What's on the Web for Mathematics Teachers?

A Web Site Review

Reviewed by Christine Longcore

Many mathematics teachers look at the World Wide Web in one of two ways. Some view it as a dizzying array of data and information that requires too much effort to navigate often. Others view it as a rich teaching resource that they use frequently. It can be argued that both viewpoints are accurate to some degree. Indeed, so many Web sites for mathematics teachers exist that a search of the World Wide Web may yield an overwhelming list of options. It is often difficult and time-consuming to discern which sites are most relevant to secondary teachers and students. For teachers who do sift through many of the choices, the Web offers mathematics problems and real-world data, as well as a way to communicate with other mathematics teachers. In order to demonstrate the rich mathematics teaching resources that are available on the World Wide Web, we selected one Web site and explored how it connects to a topic in the secondary curriculum.

http://forum.swarthmore.edu
The Math Forum
An Online Math Education Community Center
Created and maintained by Swarthmore College
Funded by the National Science Foundation

In reviewing this Web site, we wanted to find out how much material was available in a specific mathematics topic area. We chose to focus on geometry and discovered that the page suggests projects, Internet resources, and discussion groups, all related to this topic. In each of these three areas, we considered how the information relates to geometry and how teachers can use the site in their classrooms.

Projects

Students can work on a Geometry Problem of the Week, which is an interactive E-mail project, or they can visit an online exhibit of Oriental carpets. Some examples from the Problem of the Week project require students to split a parallelogram, figure out the areas of circles in a target, or determine dimensions for a hunting platform. Visitors to the Oriental carpet exhibit use the study of symmetry to analyze patterns in the carpets.

Internet Resources

Teachers will find links to other sites that provide classroom materials, more Internet projects, and discussion groups. Teachers who use dynamic, interactive geometry software share sketches and read about different ways to use the software in their teaching. Along with their students, teachers can explore topics such as perspective drawing, electronic origami, tangrams, and tessellations.

Discussion Groups

The site includes a range of discussion groups on precollege geometry, geometry research, dynamic geometry software, and geometry puzzles. For example, the precollege geometry discussion groups include questions and answers about the standard names for parts of a sphere, writing proofs, and measuring the diagonals of a cube.

Focus on Software

Besides projects, Internet resources, and discussion groups, another section of Swarthmore's Math Forum merits special mention. The Corner for Interactive Geometry Software (CIGS) is designed especially as a support resource for teachers who use dynamic, interactive geometry software, such as *Cabri Geometry* from Texas Instruments or *Geometer's Sketchpad* from Key Curriculum Press. This "corner" of the Math Forum provides treasures such as instructional activities and lessons, sketches to study or download, access to discussion groups, and opportunities to send in questions about the software. Teachers can also download demonstration versions of the software. CIGS exemplifies how the Math Forum brings together teachers with similar specific interests and concerns about teaching mathematics.

The Big Picture

The Math Forum has three major strong points. First, the site constantly provides new information. Teachers can return again and again to find updated news about educational issues, daily additions to question-and-answer forums, and fresh mathematics problems and solutions. Second, the site includes many links to other Web sites of special interest to mathematics teachers, making it easier and more efficient for teachers to find pertinent information, data, and problems. These links allow teachers to discover sites they might not find otherwise. Third, the site's interactivity boosts its value for teachers who are eager to give and receive input about what happens in their mathematics classrooms. Teachers can choose a broad focus on general mathematics education issues or narrow the focus to a particular topic, including geometry, calculus, precalculus, or algebra. The Math Forum is an ideal starting point for mathematics teachers who want to use the World Wide Web as a teaching resource but are unsure about where to begin the journey.

tech**LINK**

In **Techlink,** mathematics teachers find out about software, Web sites, calculators, and creative computer and calculator projects. If you want to share your ideas, please complete the **Suggestion Box** form on page 15.

eLc

Everyday Learning Corporation

ELC's 10th ANNIVERSARY

Help Us Celebrate the 10th Anniversary of Everyday Learning!

Everyday Learning is celebrating 10 years of working closely with elementary teachers, administrators, and students. Two years ago, we began working with secondary mathematics educators as well. During our anniversary year, we are collecting photographs and stories that will serve as a record of the people and events that have helped shape our history. We invite you to share your stories of how you have worked with Everyday Learning to implement our secondary curricula in your classrooms. Your contributions will be a meaningful addition to the record of our first 10 years. Please send your writings and photographs to Everyday Learning Corporation, attn: John Estrada, Two Prudential Plaza, Suite 1200, Chicago, IL 60601.

NEW!* HUMAN BIOLOGY *PROGRAM

Everyday Learning Introduces *Human Biology for the Middle Grades* from Stanford University

Human Biology is a unique approach to life science that can supplement or extend any science or health curriculum. Developed by the Human Biology Middle Grades Life Science Curriculum Project at Stanford University, *Human Biology* develops life science concepts geared to the needs and interests of adolescents. It fully aligns with National Science Education Life Science Standards for the middle grades. Teachers can select modules about human systems, the environment, or adolescent topics. Each of the following modules includes softbound student books and a Teacher's Guide. A *Human Biology* Program Guide for teachers is also available.

***Human Biology* Modules**

Breathing	*Lives of Cells*
Circulation	*Nervous System*
Digestion and Nutrition	*Reproduction*
Ecology	*Sexuality*
Genetics	*Your Changing Body*

To find out more about *Human Biology*, contact Customer Service at 1-800-382-7670.

COMING SOON—*EVERYDAY LEARNING MIDDLE GRADE MATHEMATICS*

Everyday Learning is collaborating with a well-known author team to develop an innovative middle school mathematics program for grades 6–8. When completed, this middle school program will be an excellent first step for students who will later use any of ELC's secondary mathematics curricula. If you would like to receive prepublication information and materials or find out how you can become a teacher reviewer, please indicate your interest by completing the form below and mailing it to Everyday Learning Corporation, attn: John Estrada, Two Prudential Plaza, Suite 1200, Chicago, IL 60601.

Name: ____________________

Position and Grade Level: ____________________

Home Address: ____________________

Home Phone: ____________________

School Name: ____________________

School Address: ____________________

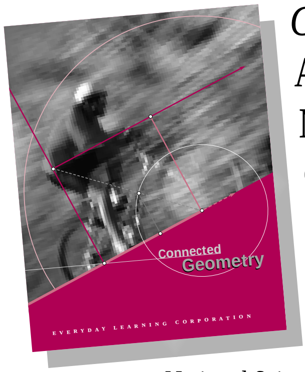
School Phone: ____________________

E-mail address: ____________________

Current middle school mathematics program: ____________________

☐ Yes, I would like to find out how to become a teacher reviewer for ELC's new middle school mathematics program.

new & revised PRODUCTS



Connected Geometry A Mathematical Habits of Mind Approach to Geometry

Connected Geometry, a new full-year geometry course, uses a mathematical habits of mind approach to teaching geometry. In this course, funded by the National Science Foundation and developed by Education Development Center Inc., students learn important mathematical ways of thinking while mastering the important ideas of geometry. *Connected Geometry* is an in-depth course in the essential ideas of geometry, not merely a survey of geometric topics. Each lesson is designed to connect active student learning and mathematical thinking to geometry content. Geometry and problem solving are connected to other fields of mathematics, other academic disciplines, and students' own experiences.

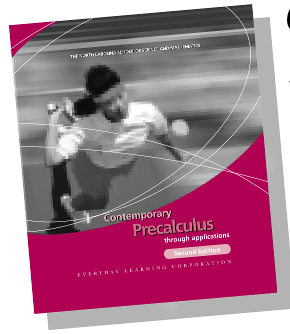
What are mathematical habits of mind?
Mathematical habits of mind are characteristic ways of thinking used by mathematicians and other effective problem solvers. By emphasizing these ways of thinking, *Connected Geometry* fosters a classroom atmosphere in which students learn to act like mathematicians. They communicate about mathematics, delve into problems, conjecture, attempt to verify their conjectures, and define their reasoning about mathematical results.

Connected Geometry emphasizes the following 10 mathematical habits of mind:

- Picturing the visible, picturing the invisible
- Looking for invariants
- Using precise language to describe and analyze
- Reasoning by continuity
- Tinkering with problems
- Using different systems
- Mixing deduction with experimentation
- Proving
- Reasoning about processes
- Analyzing and interpreting figures

The *Connected Geometry* Teacher's Resource Package includes a Student Edition, Teaching Notes, a Solution and Problem Solving Resource, Assessment Resources, Teaching Resources, and expanded versions of every unit on a CD-ROM that is compatible with both Macintosh and Windows systems.

The *Connected Geometry* CD-ROM will be available in November 1998. The *Connected Geometry* text and Teacher's Resource Package will be available January 1999. Please call Customer Service at 1-800-382-7670 for more information.



Contemporary Precalculus through Applications, Second Edition

The Second Edition of the highly successful and innovative *Contemporary Precalculus through Applications* will be available in January 1999. The authors at The North Carolina School of Science and Mathematics have revised the content, classroom-tested new lessons, and upgraded technology applications.

- An updated approach to the precalculus curriculum. Each chapter uses a "big picture" approach to meet specific learning goals. Lessons offer opportunities for open-ended exploration and discovery activities. The content also has a stronger algebraic base. The integration of data analysis provides good preparation for Advanced Placement (AP) Statistics as well as for calculus.
- Accessible and integrated technology. The Second Edition takes advantage of changes in technology by integrating use of graphing calculators into the problem-solving process. There are more real-world, easy-to-use applications of technology to mathematics content. The technology integration has made it possible to add new topics and present some topics earlier.
- Improved articulation with *Contemporary Calculus through Applications.* New chapter sections based on hands-on investigations and experiments are more closely aligned with the labs in *Contemporary Calculus.* They also provide students with a better understanding of how to build and use mathematical models. Consequently, the applications more frequently motivate students' need to learn the mathematics.

Second Edition Table of Contents

Chapter 1 *Data Analysis*
Chapter 2 *Functions*
Chapter 3 *Exponential and Logarithmic Functions*
Chapter 4 *Modeling*
Chapter 5 *Circular Functions and Trigonometry*
Chapter 6 *Combination of Functions*
Chapter 7 *Matrices*

The Student Edition will be available in January 1999. The Teacher's Resource Package includes the Student Edition, an Instructor's Guide, Assessment Resources, and a Solutions Guide. These materials will also be available in January 1999. If you are interested in ordering these revised materials, please call Customer Service at 1-800-382-7670.

suggestion box

Name ______________________ Position ______________________

School ______________________ City & State ______________________

Secondary Mathematics Program ______________________________________

Home Phone ______________________ School Phone ______________________

Check all that apply. Below is my

- ❑ Question for the **Ask the Authors** section (Please be specific.)
- ❑ **Tip Trade** idea
- ❑ **Interview** topic
- ❑ Suggestion for **Techlink**
- ❑ Suggestion for **Mathematics Bookshelf**
- ❑ Other ______________________________________

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I would also consider

- ❑ Participating in a survey or interview
- ❑ Writing for ELC's Mathlink. Topic ______________________
- ❑ Reviewing books, articles, or software
- ❑ Interviewing another teacher or administrator

ELC's Mathlink's success depends upon educators like YOU who are willing to get involved in the exchange of ideas.

Please take a moment to share your ideas, questions, answers, suggestions, thoughts, experience, advice, achievements, or insight.

Return this form to:
ELC's Mathlink
P.O. Box 812960
Chicago, IL 60681

You may also call 1-800-382-7670 with your ideas or send E-mail to Clongcore@tribune.com.

Thank you for participating in ELC's Mathlink **Suggestion Box.**

the LAST word

Share your ideas and experiences in *ELC's Mathlink*

Have you ever thought that you might like to contribute to *ELC's Mathlink* but have some questions? Below are some commonly asked questions and our responses.

Who writes the articles for ELC's Mathlink**?**
Secondary mathematics teachers and administrators write about their ideas, research, and experiences in the classroom. The curriculum authors also write articles for the newsletter. Each contributor offers a unique perspective on secondary mathematics curricula, including algebra, geometry, precalculus, calculus, and integrated mathematics; and current issues in mathematics education.

I would like to contribute an article, but I haven't written for a publication before. Will I receive input from ELC's Mathlink *editors as I write the article?*
We will be available to answer your questions and offer input on your writing from the time you propose a topic to the time you turn in that well-crafted final draft. Contributing to *ELC's Mathlink* is a valuable opportunity to write for an audience of fellow teachers. You have the option of writing an article, a book or software review, classroom management hints, or an interview. Or, if you have another idea, tell us about it.

What should I do if I want to write an article for ELC's Mathlink?
Write down your ideas and send them in! Either send in a rough draft of an article or just jot down a few notes about the topic you would like to address. Don't forget to include your name, school name, address, and phone number. E-mail addresses work, too! You can send *ELC's Mathlink* a note addressed to clongcore@tribune.com.

I want to comment on an article that was in the last issue of ELC's Mathlink. *Do you print a "Letters to the Editor" column?*
Yes, we welcome your input on the articles and features in the newsletter. You can send all letters and other submissions to Everyday Learning Corporation, attn: *ELC's Mathlink* Editor, Two Prudential Plaza, Suite 1200, Chicago, IL 60601. You may choose to use the **Suggestion Box** form on page 15 of this issue to share your ideas.

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