One of the most important and controversial problems in earth and space science today is measuring, understanding, and predicting causes of global warming. There is concern that the average annual temperature of Earth appears to have been increasing over the past century. Many scientists believe the increase is probably caused by greenhouse gases that reduce the radiation of energy from Earth’s surface into space. The graphs below show two key patterns of atmospheric change over time.

The challenge for scientists is deciding how current trends in greenhouse gas amounts and world temperature change should be projected into the future. Different projections imply that different actions are needed, each with important consequences for industry, agriculture, and personal lifestyles.
In the first two courses of *Contemporary Mathematics in Context*, you investigated several important families of functions that are useful in describing and predicting patterns of change, some of which are similar to those you considered in the “Think About This Situation” above. The members of each family have closely related patterns in tables, graphs, and symbolic rules. In this unit, you will review the properties of each function family and investigate ways to modify the basic function rules to model more complex situations.

**Modeling Atmospheric Change**

As different scientists have studied the historical records of temperature and carbon dioxide data, they’ve proposed different scenarios for the future of global warming. Each is based on certain assumptions about the best models for patterns of change.

1. Data giving Earth’s surface temperatures are collected from several sources: over 10,000 land-based weather stations, weather balloons sent up regularly by several hundred of those stations, ships and fixed buoys in the ocean, and orbiting satellites. These data are combined to estimate Earth’s annual average temperature, which is currently 60°F. The rate at which that average Earth temperature is changing is controversial. Many scientists believe it is rising, but estimates vary from about 0.05°F to 0.15°F per decade.

**INVESTIGATION 1**

Think about ways that future global warming could change Earth’s atmosphere and your own life.

a. Based on the data given in the graphs on the previous page, what strategy for projecting change in global temperature would make sense to you?

b. What strategy for projecting change in atmospheric carbon dioxide makes most sense to you?

c. If global temperature increases over the next 50 years, how do you think that change will affect our overall climate, agriculture, and personal lives?
a. Write rules for function models that predict the annual average temperature \( x \) decades from now for three different rate-of-increase estimates: 0.05, 0.10, and 0.15 degree per decade. Draw sketches of the various models on the same coordinate system.

b. Write equations or inequalities whose solutions would answer the following questions in the cases of low (0.05˚ per decade) and high (0.15˚ per decade) rate-of-change estimates. In your group, discuss how the results could be determined using function tables and graphs and by manipulation of the symbolic expressions in each rule. Then individually answer the questions, in both the low-estimate and the high-estimate cases, using the method that seems most effective.

- What will the average Earth temperature be 50 years from now?
- When will the average Earth temperature reach 61˚F?
- How long will the average Earth temperature remain below 62˚F?

2. Perhaps the most well-known theory for global warming is that the increase of atmospheric carbon dioxide (CO\(_2\)), methane, nitrous oxide, and chlorofluorocarbon gases is responsible. Roughly 60% of the increase in these gases is from carbon dioxide. The mass of CO\(_2\) in the atmosphere is commonly measured in billions of tons or gigatons (abbreviated Gt). Estimates in 2000 suggested that Earth’s atmosphere contained about 750 Gt of CO\(_2\), with another 3 Gt added each year. Atmospheric carbon dioxide is increasing because human activities send more CO\(_2\) into the atmosphere than natural biological processes remove. For example, the burning of fossil fuels sends 5 to 6 Gt of CO\(_2\) into Earth’s atmosphere every year.

a. Write a function rule to estimate atmospheric CO\(_2\) at any time \( x \) years after 2000.

b. Use the model from Part a to answer these questions:

- What level of atmospheric CO\(_2\) can we expect in the year 2020?
- When can we expect atmospheric CO\(_2\) to reach 800 Gt?

c. Suppose that when the atmospheric CO\(_2\) reaches 800 Gt, we find a way to reduce human emissions and increase biological processes that extract CO\(_2\) from the atmosphere.

- What rate of change would be required to bring atmospheric CO\(_2\) back to 2000 levels in 20 years?
- What function would predict atmospheric CO\(_2\) levels \( x \) years after the time at which corrective action began?
3. From the graph of atmospheric CO₂ reproduced at right, it looks as if, over the last 250 years, the rate of increase has not been constant. Suppose that the 2000 increase of 3 Gt per year is expressed as a percent and that future increases occur at that same percent rate.

a. What is the 2000 percent rate of increase?

b. Write a function rule to estimate atmospheric carbon dioxide \( x \) years after 2000, assuming growth from 750 Gt at a constant percent rate.

c. Compare estimates about the growth of atmospheric CO₂ over the next few decades using the model in Part b to your estimates using the model in Activity 2, which is based on different assumptions. Explain reasons why you believe one or the other model is better.

It has been estimated that in the 10,000 years since the end of the last ice age, the annual average temperature of Earth has increased by about 9°F and atmospheric carbon dioxide has increased by at least 50%. Scientists arrived at such estimates by analyzing material that has been trapped deep in very old glaciers and on the floors of lakes and oceans for thousands of years.

One of the interesting problems in such work is estimating the age of deposits that are uncovered by core drilling. A common technique is called carbon dating. Carbon occurs in all living matter in several different forms. The most common forms (carbon-12 and carbon-13) are chemically stable; the other form, carbon-14, is radioactive and decays at a rate of 1.2% per century. By measuring the proportion of carbon-14 in a scientific sample and comparing that figure to the proportion in living matter, it is possible to estimate the time when the matter in the sample stopped growing. Despite the very small amounts of carbon-14 involved (less than 0.000000001% of total carbon in living matter), modern instruments can make the required measurements.

4. Suppose that drilling into what was once a lake bottom produces a piece of wood which, according to its mass, would have contained 5 nanograms (5 billionths of a gram) of carbon-14 when the wood was alive. Use the fact that this radioactive carbon decays continuously at a rate of about 1.2% per century to analyze the sample.

a. How much of that carbon-14 would be expected to remain 1 century later? 2 centuries later? \( x \) centuries later?

b. Estimate the half-life of carbon-14.

c. What age estimate would make sense if the sample actually contained 3 nanograms of carbon-14? If it contained only 1 nanogram?
One of the ominous and spectacular predictions about global warming is that the melting of polar ice caps and expansion of ocean water will cause sea levels to rise and flood cities along all ocean shores. One estimate predicts a 1-meter rise in sea levels by the year 2100, a change that would flood large parts of low countries like the Netherlands and Bangladesh.

Estimates of such a rise in the sea level depend on measurements of glacier volumes and ocean surface areas. Earth is approximately a sphere, and oceans cover approximately 70% of Earth’s surface. The Greenland and Antarctic ice sheets cover nearly 6 million square miles and contain nearly 7 million cubic miles of ice, but that water is only 2% of all water on the planet.

5. In making estimates of the size of Earth (and other spherical planets as well), it’s useful to have formulas showing the circumference, surface area, and volume of a sphere as functions of the diameter or radius. Sometimes it’s useful to modify those relationships to show the radius or diameter required to give specified circumference, surface area, or volume.

a. Which of the following function rules will give circumference as a function of radius \( r \)? Which will give area? Volume? What clues can you use to make the correct match, even if you don’t remember the specific formulas?

- \( f(r) = 4\pi r^2 \)
- \( g(r) = \frac{4}{3}\pi r^3 \)
- \( h(r) = 2\pi r \)

b. What patterns would you expect in graphs of the functions giving circumference, surface area, and volume of a sphere?

c. Earth is not a perfect sphere, but nearly so, with average radius of about 4,000 miles. What is the approximate surface area of Earth’s oceans? What volume of water would be required to raise the level of those oceans by 3 feet? (Assume raising the level would not change the surface area of the ocean significantly.)

d. What rise in ocean levels would be caused by the total melting of the Greenland and Antarctic ice caps? (Again, assume the surface area of the oceans would not change.)

e. Earth is only the fifth largest of the planets in the solar system. The largest planet, Jupiter, has a radius roughly 11 times the radius of Earth. The radius of Mars is roughly half that of Earth. Based on these facts, how would you expect the circumference, surface area, and volume of Jupiter and of Mars to compare to the corresponding measures of Earth? Compare your answers and analysis methods to those of another group. Resolve any differences.
6. The gravitational force that holds all of us anchored to Earth’s surface diminishes as one moves up into the atmosphere. In general, the gravitational force of attraction between two masses is directly proportional to the product of the masses and inversely proportional to the square of the distance between their centers.

a. What does the above description of gravitational force suggest will happen as the distance between two planetary bodies increases? As one or both of the bodies increase in mass?

b. Which of the following expressions matches the given information about the force between masses \( m_1 \) and \( m_2 \) located at a distance \( d \) apart?

- \( F = k(m_1 m_2 - d^2) \)
- \( F = k\left(\frac{m_1 m_2}{d^2}\right) \)
- \( F = k\left(\frac{m_1 m_2}{d^2}\right)^2 \)

c. If the distance between two attracting masses is doubled, how will the gravitational force of attraction between those bodies change? What if the distance is tripled?

**Checkpoint**

In this investigation, you modeled aspects and consequences of atmospheric change with various types of functions.

a. Which of those situations (if any) involved the following families of functions?

- Linear models
- Exponential models
- Power models
- Quadratic models

b. For each of the function families listed in Part a, what general patterns do you expect in the following?

- Graphs
- Tables of values
- Symbolic rules

c. What conditions or data patterns in problem situations provide clues about the appropriateness of using each of the function models in Part a?

*Be prepared to share your ideas with the entire class.*