Think About This Situation


a. How would you describe the trends shown in the data points and the linear models that have been drawn to match patterns in those points?

b. Why do you suppose the percent of women doctors has been increasing over the past 40 years?

c. Would you expect the trend in the graph to continue 10 or 20 years into the future? This means 10 or 20 years beyond 2000.

d. How would you go about finding equations for linear models of the data trends?

e. If you were asked to make a report on future prospects for numbers of male and female doctors, what kinds of questions could you answer using the linear models?
INVESTIGATION 1: Using Tables and Graphs

There are several kinds of questions that occur naturally in thinking about the trends in numbers of male and female medical doctors. To plan for future educational programs and medical services, medical schools, hospitals, and clinics might wonder:

- When will the number of female doctors reach a 40% share?
- When will the numbers of male and female doctors be equal?
- How long will the number of male doctors remain above a 70% share?

The trends in percent of male and female medical doctors can be modeled by the following related linear equations.

Percent of Male Doctors: \( Y_1 = 98 - 0.54X \)
Percent of Female Doctors: \( Y_2 = 2 + 0.54X \)

Here, \( X \) stands for years since 1960. \( Y_1 \) and \( Y_2 \) stand for percent of all U.S. medical doctors. Using symbolic models, the three prediction questions above can be written as algebraic equations and inequalities:

- \( 40 = 2 + 0.54X \)
- \( 98 - 0.54X = 2 + 0.54X \)
- \( 98 - 0.54X > 70 \)

The problem is finding values of \( X \) (years since 1960) when the various share conditions hold.

1. Write equations and inequalities that can be used to answer each of these questions about the relation between numbers of male and female medical doctors in the United States.
   a. When might the share of male doctors fall to 40%?
   b. How long will the share of female doctors remain below 60%?
   c. When will the number of male doctors be double the number of female doctors?

2. Write questions matching each of these equations and inequalities.
   a. \( 98 - 0.54X = 65 \)
   b. \( 2 + 0.54X < 30 \)
   c. \( 98 - 0.54X = 4(2 + 0.54X) \)
Writing equations and inequalities to match important questions is only the first task in solving the problems they represent. The essential next step is to solve the equations or solve the inequalities. That is, find values of the variables that satisfy the conditions.

One way to solve equations and inequalities matching questions about numbers of male and female medical doctors is to make and study tables and graphs of the prediction models.

<table>
<thead>
<tr>
<th>X</th>
<th>Y₁</th>
<th>Y₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>98</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>92.6</td>
<td>7.4</td>
</tr>
<tr>
<td>20</td>
<td>87.2</td>
<td>12.8</td>
</tr>
<tr>
<td>30</td>
<td>81.8</td>
<td>18.2</td>
</tr>
<tr>
<td>40</td>
<td>76.4</td>
<td>23.6</td>
</tr>
<tr>
<td>50</td>
<td>71</td>
<td>29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X</th>
<th>Y₁</th>
<th>Y₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>65.6</td>
<td>34.4</td>
</tr>
<tr>
<td>70</td>
<td>60.2</td>
<td>39.8</td>
</tr>
<tr>
<td>80</td>
<td>54.8</td>
<td>45.2</td>
</tr>
<tr>
<td>90</td>
<td>49.4</td>
<td>50.6</td>
</tr>
<tr>
<td>100</td>
<td>44</td>
<td>56</td>
</tr>
</tbody>
</table>

3. Solve each of these equations and inequalities by finding the value or range of values of \( X \) that satisfy the given conditions. Then explain what each solution tells about prospects for male and female percents of all U.S. medical doctors. Explain how the solutions can be found (or at least estimated) in tables and graphs of \( Y₁ = 98 - 0.54X \) and \( Y₂ = 2 + 0.54X \).

a. \( 70 = 2 + 0.54X \)

b. \( 98 - 0.54X = 2 + 0.54X \)

c. \( 98 - 0.54X > 80 \)

d. \( 98 - 0.54X = 65 \)

e. \( 2 + 0.54X < 40 \)

f. \( 98 - 0.54X = 4(2 + 0.54X) \) [Hint: Consider \( Y₃ = 4Y₂ \).]

g. \( 98 - 0.54X = 1.5(2 + 0.54X) \) [Hint: Consider \( Y₄ = 1.5Y₂ \).]
4. Write and solve (as accurately as possible) equations and inequalities to answer each of the following questions about male and female medical doctors in the United States, based on the linear models suggested by recent data. In each case, explain how you can use tables and graphs of linear models to find solutions.
   a. When will the percent of male doctors decline to only 55%?
   b. When will the percent of female doctors reach 35%?
   c. How long will the percent of male doctors be above 40%?
   d. When will the percent of male doctors be less than the percent of female doctors?

5. When you solve an equation or inequality, it is always a good idea to check the solution you find. If someone told you that the solution to $45 = 98 - 0.54X$ is $X = 10$, would you believe it? How could you check the suggestion, without using either the table or the graph of $Y_1 = 98 - 0.54X$?

6. If someone told you that the solution to $2 + 0.54X \leq 45$ is $X \leq 70$, how could you check the suggestion:
   a. In a table?
   b. In a graph?
   c. Without using either a table or a graph?

7. What percents of male and female medical doctors in the United States do the models predict for the year 2020 (60 years from 1960)? How confident would you be of such a prediction?

---

Checkpoint

Many important questions about linear models require solution of linear equations or inequalities, such as $50 = 23 + 5.2X$ or $45 - 3.5X < 25$.

a. What does it mean to solve an equation or inequality?

b. How do you check a solution?

c. How could you use tables and graphs of linear models to solve the following equation and inequality:
   - $50 = 23 + 5.2X$?
   - $45 - 3.5X < 25$?

*Be prepared to share your ideas with the class.*
Bronco Electronics is a regional distributor of graphing calculators. When an order is received, a shipping company packs the calculators in a box. They place the box on a scale which automatically finds the shipping cost. The shipping cost \( C \) is a function of the number \( N \) of calculators in the box, with rule \( C = 4.95 + 1.25N \).

Use your graphing calculator or computer to make a table and a graph showing the relation between number of calculators and shipping cost. Include information for 0 to 20 calculators. Use the table and graph to answer the following questions:

a. How much would it cost to ship an empty box? How is that information shown in the table, the graph, and the cost rule?

b. How much does a single calculator add to the cost of shipping a box? How is that information shown in the table, the graph, and the cost rule?

c. Write and solve equations and inequalities to answer the following questions about Bronco Electronics shipping costs.

   - If the shipping cost is $17.45, how many calculators are in the box?
   - How many calculators can be shipped if the cost is to be held below $25?
   - What is the cost of shipping eight calculators?

d. What questions about shipping costs could be answered using the following equation and inequality?

\[
27.45 = 4.95 + 1.25N
\]

\[
4.95 + 1.25N \leq 10
\]

e. Bronco Electronics got an offer from a different shipping company. The new company would charge based on the rule \( C = 7.45 + 1.00N \). Write and solve equations or inequalities to answer the following questions:

   - For what number of calculators in a box will the two shippers make the same charge?
   - For what number of calculators in a box will the new shipping company’s offer be more economical for Bronco Electronics?
1. Parents often weigh their child at regular intervals during the first several months after birth. The data usually can be modeled well with a line. For example, the rule \( y = 96 + 2.1x \) gives the relationship between weight in ounces and age in days for Rachel.

a. How much did Rachel weigh at birth?

b. Make a table and a graph of this equation for \( X_{\text{min}} = 0 \) to \( X_{\text{max}} = 90 \).

c. For each equation or inequality below:

   - Write a question about the infant’s age and weight that the equation or inequality could help answer.
   - Use the table or graph to solve the equation or inequality and then answer your question.

   i. \( y = 96 + 2.1(10) \)
   
   ii. \( 159 = 96 + 2.1x \)
   
   iii. \( 264 = 96 + 2.1x \)
   
   iv. \( 96 + 2.1x \leq 201 \)

2. A concession stand at the Ann Arbor Art Fair sells soft drinks in paper cups that are filled by a dispensing machine. There is a gauge on the machine that shows the amount of each drink left in the supply tank. Wendy collected the following data one day.

<table>
<thead>
<tr>
<th>Number of Drinks Sold</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ounces of Soft Drink Left</td>
<td>2,500</td>
<td>2,030</td>
<td>1,525</td>
<td>1,000</td>
<td>540</td>
<td>20</td>
</tr>
</tbody>
</table>

a. Plot these data and draw a line that fits the pattern relating the number \( N \) of drinks and the ounces \( L \) of soft drink left.

b. Find the slope of the linear model. Explain what it tells about the soft drink business at the art fair.

c. Find the coordinates of the point where the linear model crosses the vertical axis. What do the coordinates tell about the soft drink business?

d. Find an equation for the linear model relating \( N \) and \( L \).

e. Write calculations, equations, or inequalities that can be used to answer each of the following questions. Answer the questions. Show how the answers can be found on the graph.
About how many drinks should have been sold when the machine had 1,200 ounces left in the tank?

How many ounces were left in the tank when 125 drinks had been sold?

How many drinks were sold before the amount left fell below 1,750 ounces?

3. Recall (from page 188) that Mary and Jeff Jordan each sell programs at the local baseball park. They are paid $10 per game and $0.25 per program sold.
   a. Write a rule relating number $X$ of programs sold and pay earned $Y$.
   b. Write equations, inequalities, or calculations that can be used to answer each of the following questions:
      - How many programs does Jeff need to sell to earn $25 per game?
      - How much will Mary earn if she sells 75 programs?
      - How many programs does Jeff need to sell to earn at least $35 per game?
   c. Produce a table and a graph of the relation between sales and pay from which the questions in Part b can be answered. Show on the graph and in the table how the answers can be found. Find the answers.

4. Emily works as a waitress at Pietro’s Restaurant. The restaurant owners have a policy of automatically adding a 15% tip on all customers’ bills as a courtesy to their waitresses and waiters. Emily works the 4 P.M. to 10 P.M. shift. She is paid $15 per shift plus tips.
   a. Write an equation to model Emily’s evening wage $W$ based on the total $B$ of her customers’ bills. Use your graphing calculator or computer software to produce a table and a graph of this relation.
   b. If the customers’ bills total $110, what calculation will give Emily’s wage for the evening?
   c. If Emily’s wage last night was $47, write an equation showing the total for her customers’ bills. Solve the equation.
   d. What is the smallest amount that Emily could make in an evening? Which point on the graph represents that amount? How is her smallest amount reflected in the equation?
   e. After six months at Pietro’s, Emily will receive a raise to $17 per shift. She will continue to receive 15% tips.
      - Write an equation to model her new wage. Graph it along with her previous wage equation in the same viewing window.
      - Compare the smallest amounts Emily could earn for each wage scale and compare the rates of change in the wages. Describe the similarities and differences in the equations and graphs.
Organizing

1. Suppose two variables \( x \) and \( y \) are related by the rule \( y = 4 - 0.5x \). Make a table and a graph of this relation for \( X_{\text{min}} = -20 \) to \( X_{\text{max}} = 20 \). Use the table and graph to solve each equation or inequality below.

   a. \( y = 4 - 0.5(12) \)  
   b. \( -1 = 4 - 0.5x \)  
   c. \( -5 = 4 - 0.5x \)  
   d. \( 4 - 0.5x \geq 0 \)

2. Graph the two equations \( y = 2 + 0.25x \) and \( y = -8 + 1.5x \) for \( X_{\text{min}} = -5 \) to \( X_{\text{max}} = 10 \). Use those graphs to solve these equations and inequalities.

   a. \( 2 + 0.25x = -8 + 1.5x \)  
   b. \( 2 + 0.25x \geq -8 + 1.5x \)  
   c. \( 2 + 0.25x \leq -8 + 1.5x \)  
   d. \( -8 + 1.5x \geq 0 \)

3. For any linear relation with rule of the form \( y = a + bx \):

   a. Explain, with sketches, how to solve equations of the form \( c = a + bx \) using the graph of the linear model.
   b. Explain, with sketches, how to solve inequalities of the form \( c \leq a + bx \) using the graph of the linear model.
   c. Explain, with sketches, how to solve equations of the form \( a + bx = c + dx \) using graphs of linear models.
   d. Explain, with sketches, how to solve inequalities of the form \( a + bx \leq c + dx \) using graphs of linear models.

4. The linear equations you have solved in this investigation have been of two forms.

   a. How many different values of \( x \) can be found that satisfy any specific equation of the form \( c = a + bx \)? Explain your answer with sketches of linear models.
   b. How many different values of \( x \) can be found that satisfy any specific equation of the form \( a + bx = c + dx \)? Explain your answer with sketches of linear models.

5. The linear inequalities you have solved in this investigation have been of two forms.

   a. How many different values of \( x \) can be found that satisfy any specific inequality of the form \( c \leq a + bx \)? Explain your answer with sketches of linear models.
   b. How many different values of \( x \) can be found that satisfy any specific inequality of the form \( a + bx \leq c + dx \)? Explain your answer with sketches of linear models.
Reflecting

1. To study a linear model like \( y = 125 + 35x \), would you prefer to use a graph or a table of this relation? Give reasons for your preference.

2. Describe a problem situation which could be modeled by the equation \( y = 10 + 4.35x \).
   a. What would solving \( 109 \geq 10 + 4.35x \) mean in your situation?
   b. Solve \( 109 \geq 10 + 4.35x \).

3. Describe a strategy for solving inequalities of the form \( c \leq a + bx \) using the table-building capability of your graphing calculator or computer software. How would you modify your strategy in order to solve inequalities of the form \( c > a + bx \) ?

4. When solving equations or inequalities that model real situations, why is it important not only to check the solution, but also to check if the solution makes sense? Show your reasoning with an example.

Extending

1. The diagram at the right shows graphs of two relations between variables:
   \[ y = x + 3 \quad \text{and} \quad y = x^2 - 3 \]
   Reproduce that diagram on your graphing calculator or computer. Use the trace function to solve the equation \( x + 3 = x^2 - 3 \).

2. Use the graph from Extending Task 1 to solve each inequality:
   a. \( x + 3 \geq x^2 - 3 \)
   b. \( x + 3 < x^2 - 3 \)

3. Refer to Task 4 of the Modeling section. A new policy at the restaurant requires wait staff to share their tips with busers. Wait staff will receive $20 per shift plus 10% in tips. Busers will receive $25 per shift plus 5% in tips.
   a. Write one equation to model the busers’ wages, and write another equation to model the wait staff’s wages. Graph these two equations on the same coordinate axes.
   b. Write and answer three questions about the wages for busers and wait staff. Based on your analysis of the graphs, write equations or inequalities corresponding to your questions.